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# STATISTICS

AND THEIR

# APPLICATION TO COMMERCE

BY

A. LESTER BODDINGTON, F.S.S.

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# STATISTICS AND THEIR APPLICATION TO COMMERCE

BY

A. LESTER BODDINGTON

FELLOW OF THE ROYAL STATISTICAL SOCIETY,
FELLOW OF THE ROYAL ECONOMIC SOCIETY

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# PREFACE.

THE value of Statistics has long been recognised in connection with Social and Economic investigation and in certain specialised branches of Commerce, such as Banking and Insurance; but there has been no serious attempt to link up the science of Statistics with Commerce.

This little Manual has been prepared to meet the requirements of Business men and to promote interest in the maintenance and use of Statistical records in commercial undertakings, so that they may form part of the machinery of scientific management.

The book does not pretend to solve the many problems which confront the business man, as many of these are clearly peculiar to each individual case, but is intended as a guide to the methods to be used by those who may be interested in the subject.

A. LESTER BODDINGTON.

LONDON, 1921.

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# INTRODUCTION.

THE business man of to-day is met by many problems which did not confront his predecessors, and chief among them is the tendency of the present day to limit by artificial means the rate of profit which he may earn. In order, therefore, to show increasing profits from his business he is compelled either to (*a*) increase his Turnover, or (*b*) reduce his expenses and cut out all waste. The former is, of course, possible, but the ever increasing competition often means that additional turnover is only obtained by increasing the expenditure to an extent which is not justified by the result. The latter is of great importance, and applies even in those cases where turnover is increasing —in short, Scientific Management is to-day the only hope of showing better profits. Waste, whether of material and/or labour must be eliminated, preventable losses must not take place, the law of increasing returns applied to its fullest extent, efficient machinery must be introduced to save time and labour wherever possible. Operating costs, distributing costs and general expenses must all be watched most closely, since an increase in any of these components may result in the profits being swept away, and in a good year's business showing a loss.

We are seeing up-to-date machinery and time-saving methods gradually introduced throughout our offices, but many heads of business houses are apt to regard the purchase price of such machinery and appliances as an expenditure, whereas it is in reality a first-class investment. This point of view is probably the cause of our business men being regarded as conversative in their methods.

Business is a science, and the office may therefore be regarded as the laboratory, and as such it should be efficiently fitted to carry out the work for which it is intended. In order to succeed in any business to-day the business man must study all the factors which enter into the production, buying and selling, exporting and importing of the goods in which he deals. He must know the right time to buy, the right price to pay, the quantity to buy, and the correct place to buy it. He must know when to place the article upon the market, the price which he can obtain, the people to whom it is to be sold. He must study transport facilities, and investigate and compare the costs of the various methods of transport which may be adopted, not overlooking the importance and convenience of speedy and certain deliveries. He must know how to create a market for a new product, or how to stimulate the demand for his goods. Financial conditions need close and careful study in order that he may know when it is advisable to expand and contract credit, or when it may pay him to purchase his goods and pay for them by raising a loan. If he is an importer or exporter, he must know the influences which affect foreign exchanges, so that he may pay or collect his debts at the most propitious moment. He must know when he may speculate with a reasonable expectation of profit, and must be able to come to an immediate decision with regard to any proposition which may be placed before him. This calls for efficiency of a very high order.

The time has come when old-fashioned methods must be scrapped, old ideas swept away, and new and improved methods used in order that business may expand, profits increase, and, in brief, development be rendered possible.

During the past few years we have seen great advances made in business organisation and methods. Where formerly the personal influence of the proprietor attracted business, the present tendency is of an impersonal nature. Buyers purchase their commodities in the best market, irrespective of the person with whom they are dealing. Small businesses, where the proprietors knew their regular customers, are passing away. Mutiple shops and Departmental Stores are springing up. Amalgamations are rife, and large businesses are increasing in number. As a consequence of this tendency the principal must rely upon others for the specialist and detail work necessary, and is obliged to devote his energies to directing the general policy. He can be greatly aided by being supplied with statistical records, which will enable him to analyse the situation and obtain a panoramic view of the business as a whole. He will thus supplement his own abilities by making use of scientific methods, will obtain the true perspective of the activities of his subordinates, and gauge the possibilities of profit-making opportunities, and thus be materially assisted in building a successful and profitable business.

# STATISTICS AND THEIR APPLICATION TO COMMERCE

## *PART I.*

## THE COMPILATION OF STATISTICAL DATA.

### SYNOPSIS TO CHAPTER I.

### BUSINESS STATISTICS AND THEIR IMPORTANCE.

§ 1.—Estimates and Their Uses.

2.—The Budget and Its Compilation.

  (*a*) Why Results differ from the Estimates.

3.—Statistics in Commerce.

4.—Objects of Statistics.

5.—The Theory of Probabilities in Commerce.

## CHAPTER I.

# BUSINESS STATISTICS AND THEIR IMPORTANCE.

Every business man finds it necessary to keep a record of his transactions, but frequently these records consist only of those financial aspects of his activities which enable him to prepare proper accounts periodically, and thus many advantages which accrue to those who use the information accumulated from time to time for placing their business affairs upon a scientific basis are not obtained.

### § 1.—Estimates and Their Uses.

Financial Institutions such as Banks and Insurance Companies can only carry on their businesses by the intelligent utilisation of past experience. As a result of studying previous records a Banker knows what cash reserve it is necessary to maintain in order to meet normal calls made upon him, and moreover can vary these Reserves as the needs of the occasion demands. In an agricultural district, for example, it will be necessary to increase the amount of cash in hand at Harvest time in order to meet the demand for money wherewith to pay the wages of the additional men required at this time. Insurance Companies base their premiums upon the results of the risks covered over a large number of years, or from other data at their disposal. Railway Companies, in order to provide for the great crowds which habitually travel at holiday times, must estimate the number of persons they may be called upon to carry and the distance and direction they will travel. These estimates are naturally based upon what has happened in the past, and thus we find additional trains provided on some services and a curtailed service on other sections of the line. While it is true that the estimates may be wrong, the system of utilising past experience has much to recommend it, since the error will be in large measure due to factors outside the control of the company, such as bad weather or a change in popular taste. In America we find many operators in futures. They base their speculative operations on the possible crops of any particular commodity in which they are interested, and their estimates of the quantity which will be available are obtained by taking the average yield per acre over a large number of years, and multiplying by the number of acres which it is estimated are under cultivation. The Government publishes much information as to yield, while the acreage is estimated by the commercial bureaux and Newspapers from the information collected by their correspondents. Such estimates, like those of the Railways, may be incorrect owing to the fact that natural forces come into play, but the estimates of the yield will be found to be approximately correct in a year which is an average one, that is, one in which no abnormal influences affect the crops. If natural factors could be eliminated errors would almost disappear. To take an example, the world's output of gold can be estimated almost exactly, since the only factor which will affect the output to any extent is the discovery of a new goldfield.

## § 2.—The Budget and Its Compilation.

The Budget which is now awaited so eagerly in these days of heavy taxation would be impossible of production were it not for the work of the Statistical Sections of the various Departments concerned. These Departments render statements showing the population, and by a comparison with previous records are able to estimate the number of people who will be called upon to pay Income Tax, while the average amount of taxable incomes will enable an estimate to be made of the yield of an Income Tax at a certain figure. In this connection it should be noted that the Statistical Departments have at their disposal facts which enable a far more accurate result to be obtained than would otherwise be the case, in so far that such a large proportion of Income Tax is collected under Schedule D (in the fiscal year 1916–17 the total income from which tax was received was £982 millions, and of this £594 millions came under Schedule D), and the Income taxable under this Schedule is the average of three years, thus the Revenue Authorities are already in possession of two-thirds of the information. Figures as to the importation of goods are also available, and hence the yield of any additional Tax upon any commodity imported can easily be ascertained, although allowance has to be made for the fact that consumption may decrease when an increase of price takes place. In the case of Estate Duties the average over a large number of years is the only safe basis to assume since so many influences may affect the estimate.

### (a) Why Results differ from the Estimates.

It may, of course, be argued that the Chancellor of the Exchequer seldom balances his receipts and expenditure, having either a deficit or a surplus, and since this is so, that the science of statistics has not proved of much avail. It must, however, be borne in mind that the Budget is merely a collection of estimates of both receipts and expenditure. Receipts may fall from a variety of causes, such as decreased consumption of Taxable commodities owing to the increase in the price necessitated by the increased taxation ; and while the average death rate may be maintained, the estates of the deceased persons may be small, and the Exchequer suffer from the smaller yield in respect of Estate, Legacy and Succession Duties. On the other hand, the expenditure of the various Departments may exceed their estimates, or some severe and unforeseen crisis arise which necessitates additional expenditure on some branch of the activities of the Government.

## § 3.—Statistics in Commerce.

In the realms of business Statistics have not been applied except to specialised branches, and even then are largely confined to the financial side, but there seems no reason why the science should not be applied with advantage to all branches of commercial activity. A commencement has been made in Cost Accounting, which at its base is the utilisation of the information recorded of past work done, and upon such information present or future work may be based.

## § 4.—Objects of Statistics.

One of the objects of Statistics is to supply a panoramic view of a large mass of facts accumulated from past experience, to co-ordinate and condense these facts into a form easily comprehensible to the man in the street, and to use the knowledge so obtained to assist present or future operations. If this definition of the object be admitted, then the usefulness of Statistics to Commerce cannot be questioned.

## § 5.—The Theory of Probabilities in Commerce.

It is true that Statistics is a science of probabilities, whereas Accounting, another science of great importance to a business man, is an exact science, and thus since the business man sees the results of his business activities in the figures compiled by his Accounts Department he is apt to regard business as an exact science also. A moment's reflection will at once make it clear that business itself is an application of the theory of probabilities, since the business man buys or produces his goods on the probability of the demand being of such a magnitude that he will be able to dispose of his goods at a profit. The size of his order, or the quantity which he will produce, will depend upon his estimate of the probable size of the demand from his customers. He will willingly undertake to pay a high price for the commodities in which he deals if he is of the opinion that he will be able to obtain a still higher price from his customers. In other words, he estimates the demand and then buys or produces accordingly. His estimate may be incorrect. He may overstock himself, and have either to keep unsaleable stock on his shelves or else sell for less than cost, or on the other hand he may under-estimate the demand and find that his supply is exhausted before the demand is supplied, and thus he loses an opportunity of making profit. A contractor may estimate the cost of the work and enter into a contract only to find that he has made a loss owing to an error in his calculations as to the cost. On the other hand, his estimate may be too large, and thus he does not obtain the contract for which he has tendered. The successful business man is the one whose estimate most closely approaches accuracy. It may at once be said that to estimate correctly calls for skill of a very high order, and this is true, but such skill is only obtained as a result of long experience, and consequently is really the utilisation of past knowledge. This personal skill could be greatly increased if this past knowledge were carefully preserved, recorded and adequately tabulated, and such recording and tabulating is nothing less than the compilation necessary to obtain Statistics. If records are made over a long period a Standard can be obtained, and such Standard utilised as the basis of future operations ; but these operations must in turn be recorded in order that any change in conditions may be allowed to exercise its influence correctly, or anything abnormal be duly eliminated. The compilation of such records would not be an expensive process in so far that in the ordinary course of business records are made, the only additional expense being the correct tabulation and utilisation of the figures of all transactions already in existence. Care in the compilation of the records is, however, necessary, for unless they are prepared by one who is in close touch with the general policy of the business they may give a biassed view, instead of supplying that broad bird's-eye view of the business as a whole which is so essential to a busy principal. Large business houses have found it an advantage to establish special statistical and intelligence departments, which gather and co-ordinate facts and figures not only of the particular house, but of trade and finance generally. The principals then have access to data which has proved of great value when new markets are being exploited or a new venture launched.

## SYNOPSIS TO CHAPTER II.

# COMPILATION AND USES OF STATISTICS.

§ 1.—FALLACIOUS COMPARISONS.

2.—THE UNIT FOR COMPARISON.
  (*a*) Definition of Unit.
  (*b*) Division of Unit.
  (*c*) Suitable Units.

3.—SAMPLING.
  (*a*) Methods of Sampling.

4.—METHODS OF COLLECTING DATA.

5.—THE QUESTIONNAIRE.

6.—DEALING WITH THE DATA.

7.—HOMOGENEITY AND ITS IMPORTANCE.

8.—PUBLISHED DATA AND THEIR LIMITATIONS.

9.—THE TREND.

10.—TYPES OF INFORMATION TO BE OBTAINED FROM STATISTICAL RECORDS.

11.—DIVISIONS OF STATISTICAL DATA.
  (*a*) Groups.
  (*b*) Classes.
  (*c*) Series.

## CHAPTER II.

# COMPILATION AND USES OF STATISTICS

### § 1.—Fallacious Comparisons.

A business man frequently compares the result of his year's trading with the accounts of the previous period, but such comparison is to a very large extent useless, for either of the two periods under review may have been affected by abnormal phenomena and thus the perspective thrown out. The Turnover of a business may show a considerable increase, but this does not of itself prove that the business is progressing. Prices may have risen abnormally and the actual turnover in goods be less than before, or the increase may be due to price cutting in order to dispose of stocks, in which case the profit may be affected. If the profit be lower than the year before, it would not be correct to assume without full inquiry that prices had been cut, since they may have increased; but working expenses may have increased out of all proportion to the increase in the Turnover and a fall of profits result. Other influences may have affected all or any of the factors. Again, every trade has its cycles of good and bad trade, and if it should happen that a very good year was followed by a period of depression, or *vice versa,* fallacious conclusions might be drawn from a comparison of the two.

It is obvious from this that merely to compare certain of the factors will lead to errors, and if the science of Statistics is studied these errors should not be made All the factors should be studied if the real facts of the situation are to be deduced. Every unit must be considered and given its due weight, for if this be not done we shall be arguing that since one part is progressing the whole must be well. Examples of this kind of reasoning can easily be called to mind. For instance, during the early part of the year 1920 the value of our exports showed increases, and many of our newspapers forecasted a more prosperous time. They, however, omitted to take into consideration the ever increasing prices, and until quantities are available as well as the monetary value of our Exports we are unable to say if the volume of trade is increasing or not. It has also been argued that because our Export Trade is improving the country must be in a flourishing condition, but this is not necessarily so. The Export Trade is only a part of the business of the country, and frequently when the Home Trade is suffering from depression the Export Trade shows an actual increase, owing to the fact that Manufacturers and Merchants always endeavour to obtain the best price for their commodities, and if the Foreign Markets will yield a better return than is obtainable at Home, then the goods are shipped to other countries.

## § 2.—The Unit for Comparison.

Any information which we desire to use must therefore be *complete in all respects*, if we are to obtain accurate inferences, and care must also be taken to use them intelligently. It is also essential that all similar data should be compiled on exactly the same lines, as otherwise they are useless for purposes of comparison. It is obviously fallacious to compare the profits of two years if the amount of Working Capital in the two periods is not the same, nor would any end be served by comparing the profits of two periods if in one of them some extraordinary profit had been realised, say, from the realisation of some investments. If the best results are to be obtained, the profits must be arrived at after the compilation of the accounts on the same basis, *i.e.* after crediting the usual trading profits, and charging the usual items, or at any rate the same class of items.

### (a) Definition of Unit.

It is a truism to say that definitions vary greatly. Take "Profits" as an example. What are Profits? If this question be addressed to a business man his answer will vary greatly from that obtained in the reply of an Accountant, while the Inspector of Taxes or the Lawyer would each give an entirely different rendering. Each in his own meaning would be correct, but to compare the accounts compiled according to the various definitions would lead to no intelligible result for practical purposes. In Statistics the first important thing to do is to fix upon the unit which it is desired to record, carefully define it, and then proceed to compile the records, seeing that the definition is carefully adhered to. Even though the definition be at fault, the resulting comparisons will be homogeneous in their construction and their further use correct.

### (b) Division of Unit.

Unless the unit chosen is applicable to all the cases which fall under review, it will be necessary to split the unit into groups or classes in order to get more accurate results. Wages form a good example of such a unit. In its broad acceptation in the business world the word has come to mean the remuneration paid to those workers engaged in the actual handling of the commodities in which a business engages, and thus the pay of clerks, etc., fall into the category of salaries. This seems so clear that it might be thought sufficient, but in practice it will be found that unless all the workers are paid upon the same basis comparison is rendered difficult, if not useless. Where different rates are paid the workers should be grouped according to the basis of pay. As an example let us consider a case where both male and female labour are employed, and assume, as is usually the case, that the rate for women is lower than that for men. If any change in the composition of the operating force takes place, such as frequently happened during the War, when male labour was reduced and female labour greatly increased, the average wages paid to each worker would, all other things being equal, fall, unless the wages were split into the two groups: "Wages paid to Men" and "Wages paid to Women," in which case any change in the relative numbers employed will not change the average. Take a concrete case. A firm employs 100 men and 500 women, and pays to the former a total of £500 and to the latter £1,000 per week, then the average wage per man is $\frac{500}{100}$, or £5 per week, and per woman $\frac{1{,}000}{500}$, or £2 per week. If the pay roll is combined, there are 600 employees earning £1,500, an average of £2·5 per week, an average which does not represent the true state of affairs. Now, suppose the number of men employed by the firm to fall to 50, with a weekly pay roll of £300, and the number of women becomes 1,000, with a wage bill of £2,325. The average earnings of the men is now £6 per week, and of the women £2·35 per week, an increase in the average earnings in each case. If, however, separate records have not been kept, the wage bill would be £2,625 for 1,050 persons, or an average of £2·5. If this latter fact were all the information at our disposal, we should assume that no change in the rates of pay had taken place, whereas, eliminating any question of overtime, the rates had increased considerably.

### (c) Suitable Units.

It is care in the use of the information at our disposal that will make a Statistical Department useful. It would be invidious to compare the output of a factory for a week in which a Bank Holiday occurs with that for a week in which full time was worked, nor would it assist us to compare the figures for output of two weeks when in one of them more labour was employed than in the other; but figures relating to the output of a works or plant can be compared with great advantage if we reduce the output to some unit comprised both of time and labour. Such a unit would be the "man-hour." The output per man-hour would be obtained by dividing the total output by the number of men employed, and by the total hours worked in producing such output. Such a standard will also enable accidents to be eliminated, for a breakdown will have no effect upon the output per man-hour, though of course the total output will of necessity be reduced. If the results are now tabulated in two columns, the first showing the total output over a given period, and the second the Output per "Man-Hour" for the same period, succeeding periods can then be easily compared. If the figures in the first column fall while those in the second remain constant, then attention will need to be given to the reason therefor, which may be owing to a reduction in the number of Operatives, or to the fact that the number of hours worked was lower, due to breakdown or other causes. If, however, extra columns are added showing (*c*) the number of men employed and (*d*) the total hours worked, the information necessary will then be at hand, so that any fluctuation in the figures will be explained if due to either of the causes enumerated. Should the fluctuation be found to be due to some other cause, inquiry would of necessity be made to account for the change. If the figures in the second column fall, then either the quality of the labour employed is falling off, or there is some factor which is handicapping production and which needs remedying. The following example will serve to illustrate the utility of such a statement :—

| Week Ended. | Total Output. Tons. | Output per Man-Hour. Tons. | Total Number of Operatives. | Total Number of Hours Worked. |
|---|---|---|---|---|
| January 7th .. .. .. | 4,500 | ·0025 | 200 | 9,000 |
| ,, 14th .. .. .. | 4,000 | ·002477 | 190 | 8,500 |
| ,, 21st .. .. .. | 4,400 | ·0025 | 200 | 8,800 |
| ,, 28th .. .. .. | 4,550 | ·002524 | 200 | 9,000 |
| February 4th .. .. .. | 5,000 | ·001785 | 250 | 11,200 |
| ,, 11th .. .. .. | 4,500 | ·002041 | 245 | 9,800 |
| ,, 18th .. .. .. | 5,500 | ·001808 | 260 | 11,700 |
| ,, 25th .. .. .. | 6,000 | ·001972 | 260 | 11,700 |
| March 3rd .. .. .. | 6,500 | ·002136 | 260 | 11,700 |
| ,, 10th .. .. .. | 7,000 | ·002281 | 260 | 11,800 |
| ,, 17th .. .. .. | 7,250 | ·002348 | 262 | 11,780 |
| ,, 24th .. .. .. | 7,500 | ·002430 | 262 | 11,780 |
| ,, 31st .. .. .. | 8,000 | ·002592 | 262 | 11,780 |
| Total Output for 13 weeks .. | **74,700** | .. | .. | |
| Total number of Hours worked | .. | .. | .. | **138,540** |
| Average Output per Man-Hour | .. | **·00261** | | |

By adding the figures in Column 1 and those in 4 we obtain the Total Output and the Total Number of Hours worked in the period under review, while by taking the average of Column 2 we obtain the average Output per Man-Hour for the Quarter, and this latter figure would then be the basis of comparison for succeeding weeks. At stated periods the average should be adjusted so as to include all the work of the weeks up to that date, care being taken to obtain the correct average and not merely by "averaging the averages" (*i.e.* adding the averages and dividing by the number of the averages so added). In this way the average is constantly being amended to include current working, and a more valuable standard is being obtained in so far that any abnormal influences are eliminated and a poor period is counteracted by a better one than the average, as experience shows that workpeople cannot be expected to continue a constant and steady output under varying conditions. By this Moving Average the Standard is constantly being adjusted, and consequently any improved working conditions leading to higher output are given due weight. In the statement given above it will be noticed that a very heavy fall in the production per man-hour took place during the week ended February 4th, and that subsequently, with one exception (*i.e.* the week ended February 18th), the rate has steadily improved. The abnormally small output per man-hour for the week ended February 4th may be due to one or both of two factors, both of which are revealed in the statement. The first is that during that week the number of Operatives increased from 200 to 250, and it is not only possible but probable that the new workpeople were not able to produce the normal output owing to strange working conditions. In the second place it will be seen that the average number of hours worked by operatives had dropped from 45 hours, which would appear to be the normal working week, to 44·8. It is, however, noticeable that when the force was again increased during the week ended February 18th the same phenomenon is observable, though in this case the full number of hours was worked, thus confirming the fact that Output is adversely affected when Operatives are working under strange conditions.

The utility of this table would be greatly increased were it possible to place at the head of the second column the average output per man-hour for the previous year or quarter, or whatever unit of time is adopted, thus permitting an easy comparison with previous observed results. While in the broad science of Statistics a Standard based on a long series of observations is far more reliable than that of a short term, it is doubtful if this would apply with equal force to a productive business, since working conditions are frequently altered and improved, and output affected accordingly; but if the standard be based upon the whole of the results obtained while the conditions remain unchanged, the standard will be far more reliable than one obtained over a short period of, say, a year, since the longer period will allow for and correct all extraordinary fluctuations. Mathematicians define this as the "Law of Enertia of Large Numbers." It arises from the fact that in a large mass of data phenomena are bound to occur, but can be ignored in so far that while one phenomenon will affect the result in one way another will affect the data in the opposite direction, and consequently tend to nullify the effect of the first, or entirely neutralise it. The larger the mass of figures the greater opportunity there will be for this law to operate, and hence the necessity for basing the standard on figures obtained over a long period.

While the form set out above is representative, it will be found that each business presents its own problems, and consequently the business statistician will have to study his own business carefully before he will be able to fix upon a standard satisfactory for all purposes; but once that standard has been decided upon, it should not be changed without very careful consideration, for if it be altered then the previous compilation of data will need to be "scrapped," or much time and labour expended in adapting it to the new conditions.

At present the science of Statistics is only just being appreciated as an aid to business, for up to now it has been chiefly applied to Social problems and branches of business arising out of social conditions, such as Life Insurance, a field which offers a much wider scope to a science dealing with probabilities. As has already been pointed out, the larger the mass of data or figures available the greater the degree of accuracy which can be expected when such information is used as the basis of future operations. It is, however, obvious that the ordinary business does not give so wide a scope as is advisable for following the rules laid down for social statistics. On the other hand, however, the smaller data will, or should be, more accurately compiled than those which cover a wider area, and consequently can be relied upon to a much greater extent.

## § 3.—**Sampling.**

Social investigations are made by taking representative groups and thoroughly investigating these without going to the trouble entailed in dealing with the whole of the individual cases. This method of procedure is based on the "THEORY OF PROBABILITIES," which states that if a moderately large number of items be *chosen at random* from among a very large mass, such numbers are, *on the average*, almost sure to have the same characteristics of the larger group. Some Statisticians call this principle the "LAW OF STATISTICAL REGULARITY." The main point to be borne in mind is that the examples taken must be representative of the whole body which it is desired to investigate, that is to say, every unit of the mass must have the chance of being included in those which are actually investigated. If, for instance, it is desired to ascertain the average rate of profit earned in any particular trade, and the number of businesses in such trade are so numerous that it would be impossible, or too laborious, or take too much time, or prove too expensive to apply to all of them for the requisite information to enable such average to be compiled, then by circularising a sufficiently large number of them to ensure a reliable average being arrived at a reasonable estimate of such average rate applicable throughout the trade could be made. Every business must have a chance of being included, whether it be large or small, and in order to ensure this being so perhaps the best method of selecting the representative businesses would be to draw them as is done in a lottery. The process of selection is known as "Sampling." If only large businesses were included in the inquiry the average would be fallacious in so far that a large business enjoys many advantages over a small one, such as cheaper prices when buying in large quantities, or a lower ratio of expenses to Turnover. Some businesses may get more favourable terms against payment in cash, whereas other houses may not have sufficient working capital to permit of large cash purchases, and thus suffer some disadvantage. We should not measure the stature of the British Army by the average height of the men in the Guard Regiments, nor must we measure business results by taking the large businesses only. In a general inquiry of the nature outlined above care must be taken to see that every type of district is also represented, or has the chance of representation, for conditions vary greatly and the rate of gross profit earned in one part of the country may be exceeded in another, or reduced in a third. Businesses which trade for cash only, as well as those which deal on credit, must be included, for the rate of gross profit in the latter will, of necessity, need to be higher than in the former in order to cover the risk of Bad Debts.

### (a) Methods of Sampling.

By using the Lottery method of selection of cases to represent the whole of a trade or group the Law of Probabilities gives all units, examples, classes and districts the same chance of inclusion, though other methods of sampling may be equally efficacious, such as taking every tenth unit or case in the list, or pricking the requisite number of examples from the list while blindfolded; but a method of personal selection of the representative cases is not satisfactory, for unconsciously there may be some bias present which leads to the inclusion of some examples and the omission of others.

## § 4.—Methods of Collecting Data.

Having defined the field over which the inquiries are to be spread, the next thing is to decide upon the information which it is desired to obtain, and the unit which it is desired to examine and compare. It is essential, as has been previously pointed out, that in order to obtain useful and reliable data the unit in all the cases under inspection must be arrived at in the same manner, so that if the inquiry is in regard to profits care must be taken to see that the profits are in all cases calculated in the same manner, otherwise it will be found that some people will return Gross Profit, others the Net Profit, while some will bring extraordinary profits or losses into account, and so render the results incorrect. The profits adjusted for Income Tax purposes will be found to have been returned in some cases, and divisible profits may appear in the returns of Companies. In some cases Depreciation will have been allowed for, while in others it will not have been charged. Allocations to Reserve may have been allowed for in some examples, while in others no such allocation may have been made. It will be seen from this, therefore, that it is of the utmost importance that any Schedule of Questions (or " Questionnaire," as it is technically termed) must be most carefully drawn and the unit chosen very carefully defined if mistakes, misapprehensions and misunderstandings are to be avoided. Whenever results involving calculations are required it is advisable to ask for the requisite data, so that the compiler may make the necessary calculations himself, thus ensuring uniformity in the method and the result.

## § 5.—The Questionnaire.

Assuming that it was desired to ascertain some idea as to the effect of increased wages upon output in a given industry, the following might well be the schedule of questions circulated, assuming that the articles produced were of uniform standard and size, or that the same process, where several processes are necessary to give a complete product, is referred to only. Since the form is a general one it would need adaptation to a particular business.

| | |
|---|---|
| Name of Firm : | |
| Place of Business : | |
| Description of Article Produced : | |
| Method of Production, or, if various processes are necessary, the process referred to : | |
| Date of Increase of Wages : | |
| Basis of Increase : | |
| Basis of Payment of Overtime : | |
| Method of Payment, Time or Piece : | |
| Has any alteration been made in the Method of Payment during the periods under review? If so, give details : | |
| Has there been any breakdown of machinery, or other extraordinary cause of delay, which would affect Output ? If so, state the nature of such delay, the number of hours Output was affected, and the extent of decrease : | |

| | For Period of Six Months prior to Change. | For Period of Six Months subsequent to Change. |
|---|---|---|
| 1. Number of Men employed : | | |
| 2. Total Hours worked : | | |
| 3. Total Wages paid : | | |
| 4. Number of Hours of Overtime worked : | | |
| 5. Wages paid in respect of such Overtime (included in question No. 3) : | | |
| 6. Total Output : | | |

## § 6.—Dealing with the Data.

The results obtained from the circulation of such a questionnaire would be tabulated, preferably on the basis of the Output per Man-Hour, both before and after the change in the rate of wages had taken place. This Output would be obtained in the manner which has been previously specified (*i.e.* dividing the total output by the number of men employed, and also by the number of hours worked). Comparison of the figures thus obtained would give the desired information. If the total wages paid are similarly divided by the total hours worked and by the number of men employed we shall get the average wages paid per man-hour, and comparison of the rates of wages paid in relation to the output per man-hour will be rendered easy. If overtime has been worked at a remuneration in excess of that ordinarily payable, a deduction must be made from the total wages paid, in respect of the amount paid in excess of the normal rates, if the figures are to be homogeneous and represent the standard rate of pay. Where the method of production varies separate Tables would have to be prepared for each method or process of production.

Statistical information which would be of great use to the business man is rendered difficult of compilation by the reluctance of individuals or firms to disclose information which they think will be of use to a trade competitor. This reluctance is, in a way, natural, but the benefit to be derived from properly prepared Statistics would more than outweigh the disadvantages attendant upon the disclosure of important Statistics. If the utility of Statistics were only better recognised, the formation of a Statistical Bureau under the supervision of a Chamber of Trade or Commerce where data would be collected and collated for the benefit of the business world would be of immense advantage. Such Statistics might give the output per man-hour in various classes of industry, the labour-cost per ton or other unit of production, the cost per ton-mile (*i.e.* number of tons carried multiplied by the number of miles carried) of various classes of transport, and so on, while if these again, where possible or convenient, were divided into districts, the individual producer could compile his own data on similar lines and a comparison of the figures for his own business with the averages published would furnish valuable clues as to the efficiency of his plant, labour or management, and lead to better arrangements being made to bring the business into line with the general tendency. By such means the possibilities of obtaining greater profits would be pointed out.

The Board of Trade supply much valuable material in their various returns, but as a general rule the business man either has not the time or the inclination to make a study of Blue Books. It must also be admitted that many of these figures are compiled in such a manner that they are not so useful to the business man as they might or as they should be. This has been recognised, and in the early part of 1920 a special conference of Government Statistical Officers of the British Empire was held under the auspices of the Board of Trade to consider what steps could be taken to ensure homogeneity in the returns, and uniformity in the method of compilation. Among their recommendations was one relating to the formation of an "Empire Statistical Bureau" which should be empowered to demand statistical information of public importance, and should compile and publish from time to time monographs embodying important trade statistics, such as those dealing with: Trade, Production, Transport, Communication, Population, Labour and Industries, Finance, etc. Such a Bureau should prove of great assistance to the business man, but its usefulness will be greatly impaired unless general interest is taken in the subject by the business world as a whole, since unless the statistics compiled are used for business purposes their compilation will be a useless expense and waste of time. The business man needs to study this question in the broadest possible sense, and without prejudice. He should study the returns and see what information they contain which either in their present form or with the aid of additional calculation can be used to assist in placing his business upon a scientific footing. These statistics might also open up the possibility of new fields for the employment of capital and labour, as for instance when the returns of imports show a large figure for some commodity which might, with the aid of skilful organisation, be quite as easily produced in this country. This fact has received a great deal of emphasis during the War when industries sprang up in this country to replace articles previously imported from enemy countries.

## § 7.—Homogeneity and Its Importance.

The homogeneity of any tables is absolutely essential, not only in the returns at any one time, but in the returns for the whole of the periods comprised in any one Table. If it should become necessary, as it sometimes is advisable, to change the basis of compilation, care must be taken to note the change, as otherwise fallacious observations would be made. Too many notes, however, are not advisable, for they render the table cumbersome to use, and greatly impair its utility. An example of the change of base is to be found in the Tables of " Quantities of Wheat, Wheatmeal and Flour, etc., imported into the United Kingdom," published in the " Statistical Abstract of the United Kingdom " (1919 Edition, Table 42). In 1904 and subsequent years the figures were classified according to the " Countries of Consignment," instead of under the " Countries of Shipment," as in the earlier years, a most important alteration which particularly affects those figures relating to countries which either have no ports of their own, or no regular direct shipping connections with this country, and also those countries which act as intermediaries in shipments from the first named. These latter countries will show considerable reductions in the supplies of such commodities to this country, while the former will be greatly increased. In the Table given it will be seen that the exports of such commodities from the United States apparently fell from 24 million cwts. of Wheat and 16 million cwts. of Wheatmeal and Flour in 1903 to only 7 million and 9 million cwts. respectively in 1904. On the other hand, the Argentine Republic was only given credit for supplying us with 14 million cwts. of all kinds of Wheaten commodities in 1903, but in 1904 the amount had risen to 22 million cwts. If no note were appended to the Table the figures as they stood would have caused an observer to come to the conclusion that America's production of that class of commodity was falling, and that as a consequence we were compelled to resort to the Argentine to make good the deficiency, whereas it is more than probable that the output in both countries was normal.

Statistical data should, whenever possible, be gathered by the compiler, or by those trained under and supervised by him, thus ensuring uniformity in collection, sampling and tabulating. This course is not always possible, and then recourse has to be made to the circularisation of Schedules of Questions. The form of such Questionnaire needs most careful consideration if the required information and result is to be obtained. The questions must be short and clear, easy to understand and answer, and few in number. If these points are not given attention many of the schedules will not be returned by those to whom they are addressed. When requiring the opinion of those addressed the questions should be so drafted that to the majority of them the answer " Yes " or " No " without any qualification will be possible. On no account should the questions call for information of a confidential nature, as otherwise the whole of the questionnaire will probably remain unanswered. Care must also be taken to see that the questions can raise no resentment in the mind of the average man, hence they should not be of an inquisitorial nature. They must be free from ambiguity, and correctly and clearly define the information desired.

## § 8.—**Published Data and Their Limitations.**

Frequently direct inquiry is out of the question, and recourse has therefore to be made to such published statistics and data as are available. This does not usually give such satisfactory results to the observer as the direct or personal method of compilation, since the methods used in compilation are unknown, the exact definition of the unit may be unknown, and there is usually nothing to demonstrate the homogeneity of the information. Whenever it is found that figures are unreliable they should either be discarded altogether or adjusted, if adjustment is possible. It should be remembered that a smaller number of correct figures will enable a more accurate estimate to be made than a much larger number of incorrect ones.

The ordinary business man when utilising statistics will largely confine his data and observations to the figures which are available from his own business or businesses, and this should ensure homogeneity, since once the unit has been defined it is capable of being easily maintained, and a standardised method of compiling, recording and tabulating the material developed. Use should, however, be made of any reliable statistics available from other sources.

## § 9.—**The Trend.**

A business which keeps a record of the quantities of goods of a particular nature sold year by year can get a better idea of the business *trend* (*i.e.* the technical word for " tendency "), and by basing its future production upon past experience, tend to eliminate guesswork. This method is far more accurate than depending on the monetary value of goods purchased. Prices fluctuate, or, as Economists say, the purchasing value of money alters from time to time, and consequently, if there has been any change in the prices, it would give an erroneous standard to purchase goods to the value of so many £'s sterling, when the resultant quantities may be either more or less than those of the previous periods upon which the purchase has been based.

## § 10.—Types of Information to be obtained from Statistical Records.

A business man who is working on borrowed money will, by compiling statistics as to the rates of interest ruling over various periods, discover from his records that at certain times of the year money is always cheaper than at others, hence if he plans his operations so that his borrowings are heaviest when money is cheapest he should reap additional profit from the procedure. In normal times foreign exchanges move favourably to this country at certain fairly well defined periods, so that a business man knowing this can arrange his imports so that payments are made at the time when Bills on that particular country are at a discount. Records can be compiled of delivery charges so that the cheapest and speediest method may be adopted, and fluctuations in running costs traced and corrected. The output of machines can be compiled with a view to obtaining a "Standard of Output" to act for all machines working under similar conditions. Variations from this Standard would then be disclosed, and could be inquired into with a view to adjustment, thus leading to greater efficiency from both plant and workers. Tabulated results of Advertising will show the best type of advertisement, the best medium or media in which to advertise, the district or town from which the best results are obtained, and even the best days on which to advertise.

There is no limit to the variety of purposes to which statistics can be put, but it is impossible here to do more than to indicate what can be done. The data to be collected, the object of its compilation, and the uses to which it can be put are a matter for the head of each business.

## § 11.—Divisions of Statistical Data.

Statistical Data fall into two main divisions, viz. (*a*) Groups and (*b*) Classes.

### (a) Groups.

When a large number of examples of any kind have something in common (*e.g.* Factories producing the same kind of commodity), but differ from one another in some aspect which can be measured (*e.g.* producing different grades of the same article, or different sizes of the same commodity), they together form a Group, and are capable of being tabulated in grades.

### (b) Classes.

When the characteristics of the examples are different, but such difference is not capable of being definitely measured, and thus needs a separate description (*e.g.* different methods of production), special classification becomes necessary, and these become Statistical Classes.

### (c) Series.

When records are regularly made at intervals of time of the numbers of examples which fall into either or both of the above divisions, the result is a Statistical "SERIES."

## SYNOPSIS TO CHAPTER III.

# ACCURACY AND APPROXIMATION.

§ 1.—ACCURACY.
  (*a*) Standard of Accuracy.
  (*b*) How Degrees of Accuracy may be shown.

2.—APPROXIMATION.
  (*a*) Methods of Approximation.
  (*b*) Results of Approximation on Percentages.
  (*c*) Fallacies in the Use of Percentages.
  (*d*) Ratios and Their Use.
  (*e*) Decimals and Their Advantages.

3.—ERRORS.
  (*a*) Absolute Errors.
  (*b*) Relative Errors.
  (*c*) Unbiassed Errors.
  (*d*) Biassed Errors.
  (*e*) Estimating the Error.
    (1) When the Error is Unbiassed.
    (2) When the Error is Biassed.
    (3) In an Estimated Average.
  (*f*) Effect of Errors on Ratios.

## CHAPTER III.

# ACCURACY AND APPROXIMATION.

### § 1.—Accuracy.

It has already been stated that Statistics is a science of estimates, in other words an application of the theory of probabilities. We estimate the time taken to complete a certain piece of work, and we base future estimates upon our past experience of how long a similar piece of work took. We consider it probable that we shall require a certain amount of material and labour, and make provision accordingly. We estimate the quantity of wheat, or oats or barley produced annually because there is no means of measuring either the amount produced or consumed. The number of people who are present at a function is estimated more or less correctly, but no one would know exactly how many persons were present. The company promoter estimates the amount of profit which he anticipates the Company will be able to make, basing his figures upon the past experience of the business which is being acquired, or else upon the results obtained by other companies conducting similar business. Any one of these estimates may be, and often is, erroneous, and consequently it becomes necessary to examine how far we can rely upon our estimates, and how accurate our results will be if we are obliged to rely upon approximations.

#### (a) Standard of Accuracy.

Bearing in mind that absolute accuracy is an impossibility whenever it becomes necessary to use measurement, we must lay down a definite standard of accuracy for each estimate, and endeavour to maintain it throughout similar statistical work. If it becomes necessary to ascertain the quantity of coal contained in a large dump of that very essential commodity, the contents of the dump are usually estimated after it has been measured, and this estimate if expressed in tons will probably suffice for all ordinary purposes, since in so large a quantity a few pounds or even hundredweights will make but little difference. But if a chemist is estimating the quantity of poison in his stock, then such estimate would need to be correct to the nearest grain, as otherwise a serious mistake might be made. Whenever estimates are given it is advisable, and often necessary, to indicate how far the figures can be relied upon.

### (b) How Degrees of Accuracy may be Shown.

Degrees of Accuracy may be shown in various ways, and the following are some of them :—

(*a*) The quantity of coal is 1,000 tons in round numbers.

(*b*) The quantity of coal is 1,000 tons, plus or minus an amount not exceeding half a ton.

(*c*) The quantity of coal is 1,000 ± ·5 tons. (This is the mathematical way of stating (*b*), but otherwise is the same.)

(*d*) The quantity of coal is between 999·5 and 1,000·5 tons.

(*e*) The quantity of coal is 1,000 tons correct to ·05 per cent., or ·5 per mille.

In this case, as the error is small, we might rely upon round numbers as in (*a*).

## § 2.—Approximation.

Since numerous digits are confusing to the eye, and also to the mind, it is usually found advantageous when dealing with numbers composed of many figures to express them in round numbers, even when the actual numbers are themselves available. The smaller number of digits simplifies comparison and renders calculation easier, and can generally be used unless the difference between the actual numbers is so small as to preclude the use of approximations.

### (a) Methods of Approximation.

An example will illustrate this. Columns (*a*), (*b*) and (*d*) of the following Table, comprising figures relating to the working of the Railways of the United Kingdom for the five years ended 1917, have been extracted from Table 76 of the "Statistical Abstract of the United Kingdom" (1919), and show (*a*) the Total Gross Receipts, (*b*) the Total Working Expenditure, and (*d*) the proportion which the latter bears to the former in percentages calculated to the nearest whole number. Column (*c*) has been added to show the percentage calculated correct to the second decimal place. When such calculations as these are used the last digit in each case is supposed to be correct, *i.e.* it is the nearest figure. For example, in column (*c*), Table A below, the figures in the first line should read 62·617 +, but a more correct result will be obtained when approximating by using 62·62 than 62·61, since the exact number is nearer to 62·62 than to 62·61. It is often advisable to carry the calculation to two or more places beyond the figure to which correctness is desired, as, for instance, when the percentage reads 6·509. If the percentage were being stated correct to the nearest 1 per cent., and only the figure in the first decimal place has been calculated, the observer will not be quite sure whether to read this as 6 or 7 per cent., nor will the second figure in this case assist him in arriving at a decision, whereas the third figure being 9 will cause the approximation to be made to 7 per cent., for if the third figure be 9, then on approximating to the second figure this becomes 1, and the result will then read 6·51, which being over one-half will be approximated to the next number above. Thus the number 52·2500001 when approximated to the first decimal place will become 52·3, for whenever a fraction exceeds half, no matter by how small a margin, it should be counted as a whole number. All fractions under half should be discarded (*i.e.* 52·34995 would become 52·3 when written correct to the first decimal place). When the fraction is exactly one-half it should be retained or dropped at the discretion of the compiler, though when such examples are at all numerous half of the examples should be retained, and the other half discarded, thus ensuring a more correct result in the total.

TABLE A.

TABLE SHOWING TOTAL GROSS RECEIPTS AND TOTAL WORKING EXPENDITURE OF RAILWAYS IN THE UNITED KINGDOM FOR FIVE YEARS.

|  | (*a*). | (*b*). | (*c*). | (*d*). |
|---|---|---|---|---|
| Year. | Total Gross Receipts. | Total Working Expenditure. | Proportion of Working Expenditure to Gross Receipts, calculated correct to Second Decimal Place. | Proportion of Working Expenditure to Gross Receipts correct to the nearest 1 Per Cent. |
|  | £ | £ | Per Cent. | Per Cent. |
| 1913 | 139,451,429 | 87,320,550 | 62·62 | 63 |
| 1914 | 139,098,365 | 88,173,232 | 63·39 | 63 |
| 1915 | 144,860,250 | 93,378,872 | 64·46 | 64 |
| 1916 | 154,467,914 | 102,521,053 | 66·37 | 66 |
| 1917 | 168,721,469 | 115,993,894 | 68·75 | 69 |

The figures in columns (*a*) and (*b*) are of such a magnitude that they are cumbersome to handle and difficult of comparison. This latter difficulty has been overcome by the insertion of the percentage columns, but even in order to obtain these percentages fairly long calculations are necessary, and mistakes are possible. Comparison may be rendered easier and calculations simplified were we to take round numbers, *i.e.* approximate the numbers to, say, the nearest thousand, hundred thousand, or even to the nearest million, or, if preferred, we can state the figures in millions correct to the third, second or first place as the case may be, without appreciably affecting the proportion, as is shown in the following Table, compiled from Table A, but approximated to the nearest thousand :—

TABLE B.

TABLE SHOWING DATA IN TABLE A ABOVE APPROXIMATED TO NEAREST THOUSAND.

| Year. | (*a*). | (*b*). | (*c*). | (*d*). |
|---|---|---|---|---|
| | £ | £ | Per Cent. | Per Cent. |
| 1913 | 139,451* | 87,321 | 62·62 | 63 |
| 1914 | 139,098 | 88,173 | 63·39 | 63 |
| 1915 | 144,860 | 93,379 | 64·46 | 64 |
| 1916 | 154,468 | 102,521 | 66·37 | 66 |
| 1917 | 168,721 | 115,994 | 68·75 | 69 |

**(b) Results of Approximation on Percentages.**

A comparison of the two final columns of the above Table with those of Table A will show that the approximations have not made any difference in the proportions which the figures in column (*b*) bear to those in column (*a*), thus demonstrating that in such large numbers an approximation to the nearest thousand will not in any way affect any conclusions which may be drawn from the smaller numbers of figures now used. Table C shows the same data approximated to the nearest million, and a study of the percentage calculations will show that though the percentages vary in all cases in the decimal places, yet when they are taken correct to the nearest 1 per cent., only one of them (*i.e.* the year 1916) shows any divergence from the results obtained when the actual figures were used. It should, however, be noted that percentages correct to the nearest 1 per cent. are usually considered sufficient for comparative purposes, since working to several places of decimals not only involves much work, but does nothing to simplify comparison which is the main advantage of using percentage calculations.

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* If no information as to how far the figures were correct were given in the heading of such a Table, this item would be shown as 139,451$^{000}$, thus indicating the degree of correctness of the figures.

TABLE C.

TABLE SHOWING DATA IN TABLE A ABOVE APPROXIMATED TO THE NEAREST MILLION.

| Year. | (*a*). | (*b*). | (*c*). | (*d*). |
|---|---|---|---|---|
| | £ | £ | Per Cent. | Per Cent. |
| 1913 | 139 | 87 | 62·59 | 63 |
| 1914 | 139 | 88 | 63·31 | 63 |
| 1915 | 145 | 93 | 64·14 | 64 |
| 1916 | 154 | 103 | 66·88 | 67 |
| 1917 | 169 | 116 | 68·64 | 69 |

### (c) Fallacies in the Use of Percentages.

Whenever percentages are used it is essential that the basis of the calculation of the percentage be given so that there can be no misunderstanding. In the following statement this has not been done, and consequently two results can be obtained from the data, but these results vary greatly :—

> " Wages were raised 10 per cent., lowered 15 per cent., raised 25 per cent., and lowered 20 per cent., and then raised 15 per cent. in certain years. What was the change in wages over the whole period ? "

What was the basis on which the percentages were calculated ? Was it on the original wages or on the rate of wages ruling at the date the change was made ?

(1) If the percentages were based on the original wages, then the change over the whole period would be obtained as follows:—

Let the original wages = 100.

Then (*a*) the 1st change is an increase of 10 per cent., then

$$\text{wages} = \frac{110}{100} \text{ of } 100 = 110.$$

(*b*) The 2nd change is a decrease of 15 per cent., then

$$\text{wages} = 110 - \frac{15}{100} \text{ of } 100 = 95.$$

(*c*) The 3rd change is an increase of 25 per cent., then

$$\text{wages} = 95 + \frac{25}{100} \text{ of } 100 = 120.$$

(*d*) The 4th change is a decrease of 20 per cent., then

$$\text{wages} = 120 - \frac{20}{100} \text{ of } 100 = 100.$$

(*e*) The 5th change is an increase of 15 per cent., then

$$\text{wages} = 100 + \frac{15}{100} \text{ of } 100 = 115.$$

So that the change of wages over the whole period has been an increase from 100 to 115 per cent., or a change of 15 per cent. on the basic rate.

(2) If the changes had been based on the rates of wages ruling at the time of the change, the following would be the result:—

Again let the original rate of wages = 100.

Then (*a*) the 1st change is an increase of 10 per cent., then

$$\text{wages} = \frac{110}{100} \text{ of } 100.$$

(*b*) The 2nd change is a decrease of 15 per cent., then

$$\text{wages} = \frac{85}{100} \text{ of } (a).$$

(*c*) The 3rd change is an increase of 25 per cent., then

$$\text{wages} = \frac{125}{100} \text{ of } (b).$$

(*d*) The 4th change is a decrease of 20 per cent., then

$$\text{wages} = \frac{80}{100} \text{ of } (c).$$

(*e*) The 5th change is an increase of 15 per cent., then

$$\text{wages} = \frac{115}{100} \text{ of } (d).$$

Then final wages =

$$\frac{115}{100} \text{ of } \frac{80}{100} \text{ of } \frac{125}{100} \text{ of } \frac{85}{100} \text{ of } \frac{110}{100} \text{ of } 100 = 107{\cdot}525.$$

So that wages have risen in the period under review from 100 to 107·525, a difference from the previous result of 7·475. Such variations in results would not arise were proper data always supplied.

### (d) Ratios and Their Use.

Some Statisticians strongly recommend that ratios be used instead of percentages, and this would certainly do much to eliminate mistakes which arise when percentages are used and the differing bases are not specified, since we should then state that Wages had increased in the ratio of 100 to 115 (100 : 115) rather than that Wages had increased by 15 per cent. It is advisable whenever possible to dispense with decimal points, as these are frequently either omitted or misplaced, so that in the case (2) above we should find it necessary to state the ratio as 100,000 : 107,525, rather than as 100 : 107·525.

It is not wise to trust always to percentages for comparative purposes, as the following little anecdote will amply demonstrate. Two Army Medical Officers were discussing the efficiency of inoculation against Typhoid, and one of them stated that of those patients he had inoculated 50 per cent. had subsequently contracted the disease. The other was much astonished at this statement, for, said he, " My experience is the reverse of yours, for of those cases which I inoculated only one-tenth of 1 per cent. subsequently fell victims of the fever." The two opinions were so widely divergent that further inquiries were made, and it transpired that whereas the first medico had only inoculated two men, of whom one had subsequently sickened with the fever, the second officer had treated no fewer than 1,000 men, and only one of these had contracted the disease. Apparently on the face of the statements they were capable of comparison since they were expressed as percentages, yet in reality no comparison was possible owing to the varying base of calculation. If ratios had been used, no misunderstanding would have been possible, since they would have been 1 : 2 and 1 : 1,000, these figures being obtained by reducing the actual numbers to the simplest form.

### (e) Decimals and Their Advantages.

When calculations of any kind become necessary, and fractions form a part of the numerical data with which the observer is dealing, such fractions should always be expressed as decimals, for as such they are not only easier to read, but also simplify calculation in so far that Logarithms can be used if the work is long or intricate. Useful as Logarithms are, however, it must be borne in mind that in all cases where they are used the answer is only correct to the last figure of the answer, or to the last digit when succeeded by one or more ciphers, and thus cannot be relied upon when further computation is necessary.

When approximations involving the use of decimals are used it is essential that if the figure to which correctness has been calculated is a cipher, such cipher should be shown in the same manner as if the figures were a digit. For instance, if the heights of buildings are being estimated, and one is expressed as being approximately 100 feet in height, the actual height might be anything between 99·51 feet and 100·49 feet (or 99·5 and 100·5), whereas if the height be shown as approximately 100·0 feet, then the measure of variation which is possible is between 99·95 feet and 100·05 feet, thus allowing a narrower margin of error.

Another method of approximation sometimes used is that of discarding certain digits entirely, and substituting ciphers therefor. The approximation is thus made to the next hundred, thousand or million **under** the actual number, such as reading 123,126,542 and 123,546,789 as 123 millions in each case; or, if we are dealing in monetary values, disregarding the shillings and pence, and thus reading £93 2s. 6d. and £93 19s. 6d. each as £93.

In all cases where approximate figures are used there is, of necessity, a difference between the actual numbers themselves and the estimate (*i.e.* the approximate number), and therefore if it becomes necessary that such numbers should be multiplied, divided, raised to a power, or a root extracted, the divergence will be affected accordingly.

## § 3.—Errors.

The divergence between the actual number and the estimate which is made either by approximation or by any other method, is called by Statisticians an **Error**, and care must be taken not to confuse this use of the word with that to which it more usually applies, *i.e.* a mistake.

### (a) Absolute Errors.

If we estimate the population of a town to be 100,000, whereas actually the population is 99,900, then the difference (in this case 100) between the estimate and the exact number is the Error in our estimate. Such an Error is known as an "**Absolute Error.**"

### (b) Relative Errors.

If we express the absolute error as a ratio of the estimate, we obtain what is called a "**Relative Error.**" Thus, though the absolute error in the case just quoted is 100 in an estimate of 99,900, the Relative Error is $\frac{100}{99{,}900}$ or ·001001. The Relative Error is sometimes expressed as a percentage error, and were this done in the above case the error would be said to be ·1001 per cent. of the estimate.

### (c) Unbiassed Errors.

In a case where the approximation is carried out to the nearest complete unit decided upon (*i.e.* a hundred, a thousand, etc.), the estimated number may be either greater or smaller than the actual number, and when this happens the error is said to be an "**Unbiassed**" or "**Compensating**" error, because the error may be in either direction, and thus an error in one number in one direction tends to be offset by an error in another number in the opposite direction, but there must, of necessity, be a large number of items under review before we can expect the compensation of the errors to neutralise, or even approximately neutralise one another.

### (d) Biassed Errors.

If our estimates are all known to be either over or under the actual figures, as happens when we approximate to the next complete unit below the exact number, the Error is a "**Biassed**" or "**Cumulative**" error, *i.e.* the errors are all on one side, and consequently in totalling a number of items the error becomes enlarged. The greater the number of items the greater the total error will be, as will be seen from the following Table, the numbers therein having been chosen at random, and on no preconceived plan :—

TABLE D.

TABLE SHOWING RESULTS OF BIASSED AND UNBIASSED ERRORS.

| (*a*). | (*b*). | (*c*). | (*d*). | (*e*). |
|---|---|---|---|---|
| Actual Numbers. | Numbers correct to the Nearest 1,000. | Unbiassed Absolute Error. | Numbers correct to Next 1,000 under. | Biassed Absolute Error. |
| 50,750 | 51 | + 250 | 50 | − 750 |
| 76,305 | 76 | − 305 | 76 | − 305 |
| 82,805 | 83 | + 195 | 82 | − 805 |
| 40,602 | 41 | + 398 | 40 | − 602 |
| 62,643 | 63 | + 357 | 62 | − 643 |
| 102,486 | 102 | − 486 | 102 | − 486 |
| 37,284 | 37 | − 284 | 37 | − 284 |
| 25,521 | 26 | + 479 | 25 | − 521 |
| 106,972 | 107 | + 28 | 106 | − 972 |
| 585,368 | 586 | + 632 | 580 | −5368 |
| Average of items : 65,040·9 | 65,111·1 | .. | 64,444·4 | |
| Relative Errors : | .. | ·001078 | .. | ·009255 |

It will be seen from this Table that in the case of Unbiassed Errors the Absolute Error in the Total of a number of items, each of which is subject to the same unbiassed absolute error, will tend to increase with the number of items, but the relative error present in the total will diminish with the number of items. In the above Table the absolute error in the first item, as shown in column (*c*), is 250, and the estimate is $51^{000}$ [column (*b*)], and the relative error is therefore $\frac{250}{51,000}$, or ·004901. In the totals the absolute error is 632 in an estimate of $586^{\circ\circ\circ}$, which gives us a relative error of $\frac{632}{586{,}000}$, or ·001078, a greatly decreased relative error. It will also be seen that the unbiassed absolute error is almost negligible, whereas the biassed absolute error is very material.

### (e) Estimating the Error.

#### (1).—When the Error is Unbiassed.

If the exact figures composing the number were unknown, and the only information at our disposal was that the total of the items when approximated to the nearest thousand was 586,000, it would be very desirable to know what possibility of error existed in the estimate. Each of the items comprised in the total may have an error, and this error may be anything from 0 to 499, since the number may be an exact number of thousands, and the approximate and, exact numbers thus coincide, or any number over the thousand and up to the next five hundred would cause an error varying with the number by which it exceeds the thousand, similarly with the numbers over the five hundred and under the next thousand. The most likely error, however, will be the average of these numbers 0 to 499, since it is the average type of anything which we meet with most frequently, while the extremes are seldom in evidence. The average of the successive numbers 0 to 499 will be found to be approximately 249·5, or in round numbers 250, and this latter figure is sufficiently correct for ordinary purposes. To obtain the best estimate of the Absolute Error in a Total of a number of items which have an unbiassed error we *multiply the average absolute error of the items by the square root of the number of items.* If we apply this to column (*b*) of the above Table D, we shall get $250\sqrt{9} = 250 \times 3 = 750$, so that were the actual numbers and the total unknown we could write our estimate as

586,000 ± 750, since the error may be in either direction. As we know the exact numbers in this case we are able to compare the estimate with the exact quantity. The actual error as shown in column (*c*) is 632, and our estimate of 750 is therefore fairly good. It will be found in practice that the estimated error and the actual one do not often agree, and exact results cannot therefore be expected. However carefully and scientifically we work we cannot lose sight of the fact that the estimated error is based on the probability that the average absolute error of the items is the most likely error to be made.

If we have an estimate of the total of a number of items each of which is subject to the same unbiassed error, and it is desired to ascertain what is the possible relative error of such estimate, we can obtain the best possible estimate of the relative error if we *multiply the average absolute error of the items by the square root of their number, and divide by the estimated total.* From the above Table we know the estimated total to be 586,000 and the average absolute error to be approximately 250, while the number of items is 9. The estimate of the relative error in such estimated total would therefore be :—

$$\frac{250\sqrt{9}}{586{,}000} = \cdot 00128$$

From the actual results which are known to us we can compare this result with that actually correct. This latter is found to be ·001078, so that our estimate is not far from correct. We could therefore write our estimate as 586,000 correct to ·128 per cent.

### (2).—When the Error is Biassed.

When we are approximating to the next complete unit below, as in column (*d*) of the above Table D, the total of the errors found in the items will be the absolute error of the total, and this will be observed by studying column (*e*). In this particular case the error in each item may be any number between 0 and 999, since all figures between these two extremes are equally likely to have been omitted. As the error is a biassed one due to the fact that we are discarding digits, it will increase with the number of items under observation. It is obvious that the minimum error possible is 1 × *the number of items in the Table*, and the maximum error is 999 × *the number of items*. The most likely error, however, will be the average error possible, and hence the best estimate we can make of the error in the total will be *the average absolute error of the items multiplied by the number of items*. If the average of the consecutive numbers 0 to 999 be calculated, it will be found to be approximately 499·5 (or, say, 500 in round numbers). Applying the fact and the preceding rule to the Table D above, we find that the total appearing in column (*d*) to be in error by an estimate of $499 \cdot 5 \times 9 = 4495 \cdot 5$ (or if round numbers be taken $500 \times 9 = 4{,}500$), whereas a comparison with the actual figures which are available shows the actual error of the total to be 5,368. Though the estimated error in each case is not likely to be correct, this method of computing the possible error in an estimate is very useful when it is impossible to obtain the exact figures, and we must be content to work on estimates. In the absence of any exact data, therefore, we could say that the total of the nine items when approximated to the next thousand under would be $580{,}000 + 4{,}500$, as the estimate is " short " or below the exact number in each case, while if the approximation had been made to the next thousand above the estimate would need to be written $580{,}000 - 4{,}500$, as in this case the estimate is definitely known to be in excess of the actual number.

### (3).—Errors in an Estimated Average.

Frequently we find that instead of the actual numbers and totals we have only an estimate of the average of a certain number of items which are known to be correct to a defined figure. Thus the average of the items in column (*b*) in the Table above is 65,111·1 when calculated correct to the first decimal place, and we desire to ascertain the approximate variation which there is likely to be from this average. To obtain the most probable variation we *divide the average absolute error of the items* (*i.e.* in this case 250) *by the square root of the number of items* (9), and we thus get: $\frac{250}{\sqrt{9}} = 83\cdot\dot{3}$, so that we can say the average of the items is estimated to be 65,111 ± 83. Again, as we have the actual results available we are able to compare the estimate with the actual figures. The average of column (*a*) is 65,040·9 correct to the first decimal place, and hence the error in the averages of columns (*b*) and (*a*) is actually 65,111·1 — 65,040·9, or only 70·2. It will at once be seen from this result that the absolute error of the total when divided by the number of items will give us the absolute error of the average of the numbers comprising that total (*i.e.* 632 ÷ 9 = 70·2), and also **that the unbiassed errors tend to disappear in the averages.**

The relative error of an average, when the error is an unbiassed one, can be found by dividing *the average absolute error of the items by the average of the numbers as estimated, and also by the square root of the number of items*; *e.g.* taking the figures in Table D above, the relative error of the average of column (*b*) may be estimated as:— $\frac{250}{65{,}111\sqrt{9}} = \cdot00128$, a result which agrees with the estimated relative error of the total, and it will be apparent, therefore, that the relative error of the Total and the Relative Error of the Average should agree.

### (f) Effect of Errors on Ratios.

If for the purposes of comparison we are preparing ratios between two distinct groups of data, it is essential that we should know what effect any errors in the estimates we use will have upon such ratios. In the case of Unbiassed errors it will be found that they do not usually affect the ratios at all, while Biassed errors tend to disappear in the ratios. This may be illustrated in the following figures :—

| | Actual Numbers to be Compared. | | Numbers Approximated to Nearest 1,000. | | Numbers Approximated to Next 1,000 below. | |
|---|---|---|---|---|---|---|
| | 564,584 | 1,693,752 | 565 | 1,694 | 564 | 1,693 |
| Ratios: | 1 : 3 | | 1 : 2·99999 | | 1 : 3·002 | |

## SYNOPSIS TO CHAPTER IV.

# TYPES AND AVERAGES.

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2.—THE ARITHMETIC AVERAGE.
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3.—THE WEIGHTED ARITHMETIC AVERAGE.
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4.—DESCRIPTIVE AVERAGES.

5.—MEDIAN.
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6.—MODE.
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7.—GEOMETRIC AVERAGE.

## CHAPTER IV.

# TYPES AND AVERAGES.

WHENEVER we desire to form a standard unit for the purposes of comparison, such standard should always be obtained by eliminating as far as possible, or at least nullifying the effect of, any abnormal fluctuations which may have taken place, and thus be present in the data which it is desired to review. This may best be done by the judicious use of an Average, since, by combining those years which show extraordinary phenomena with those of normal years and obtaining an average spread over a long period, we eliminate to a very large degree the effects of an abnormal period, and obtain a more useful standard for comparison, since it must be recognised that abnormalities are bound to occur from time to time.

## § 1.—Use of Averages.

Averages can be used to give us a definite idea of some large group or groups of numerical data, and so enable us to compare different groups and classes which are of such magnitude in themselves that comparison unit by unit would either be impossible, or take too much time. Moreover, we can, by the careful use of averages, often obtain a reliable guide to a much greater group or class by the use of samples. Averages become compulsory when we desire to obtain an idea of the mathematical relationship existing between different classes or groups.

## § 2.—The Arithmetic Average.

In Statistics we use various kinds of averages, each of which is particularly applicable to certain fields of investigation, but some of these averages are really types. The most common kind of average, and one which has so far been referred to in this manual, is the **Arithmetic Average.** The sum or total of a number of items is known as the AGGREGATE. *The Arithmetic Average is obtained by dividing the aggregate of a series of items by the number of items in the series,* a computation which involves no particular mathematical skill.

### a) Short Method of Computation.

When, however, we are dealing with a large number of items, all of which are of nearly the same magnitude, the following "Short-Cut" method of finding the Arithmetic Average may frequently be used with advantage:—

TABLE E.

EXAMPLE OF SHORT METHOD OF COMPUTING ARITHMETIC AVERAGE.

| Items to be Averaged. | Assumed Average. | Deviations of the Actual Numbers from the Assumed Average. |
|---|---|---|
| 1684 | 1680 | + 4 |
| 1680 | 1680 | 0 |
| 1686 | 1680 | + 6 |
| 1675 | 1680 | − 5 |
| 1673 | 1680 | − 7 |
| 1670 | 1680 | − 10 |
| 1677 | 1680 | − 3 |
| 1676 | 1680 | − 4 |
| 1682 | 1680 | + 2 |
| 1683 | 1680 | + 3 |

Sum of the deviations .. .. − 14

Average deviation $= \frac{-14}{10} = -1{\cdot}4.$

Then average of the items $= 1680 + (-1{\cdot}4) =$ **1678·6.**

Explanation : take an arbitrary figure and assume this to be the average of the numbers under review, and then tabulate the deviations of the actual numbers from such assumed average. These deviations will be added, paying due regard to the signs which precede them, and will then be divided by the number of items. The result will be added to the assumed average, and the answer will give us the true average.

The Arithmetic Average is very easy of computation, and there is no need to arrange the data in any particular order before commencing the work of finding it, as is necessary with some other kinds of averages ; but, unlike some of these others, it cannot be located by inspection, unless the data are very simple, which seldom happens in Statistics.

**(b) Advantages and Disadvantages of the Arithmetic System.**

Any extremes which appear in the data will affect the Arithmetic Average, and though this is desirable in many cases, it is not always so if the average is to represent the true state of affairs, as, for instance, when calculating the average income of a district where the bulk of the population belong to the artisan class, but where a millionaire resides. Unless the number of inhabitants was very large the income of the millionaire would greatly affect the average income of the district and give a fallacious average. Every item, however small, has an influence in fixing the Arithmetic Average, and no one section of the whole can unduly influence it, but it must be borne in mind that its accuracy is greatly impaired, unless the whole of the cases are brought into consideration. It may, however, be obtained when only the number of the items and their aggregate are known, and no details of the various items are available. If, for instance, we know the amount raised by taxation in any country, and the number of the population of that country, we can calculate the average amount of Tax contributed per head of the population, although we do not know the actual amount paid by any one individual. A serious disadvantage of this kind of average is that the result we obtain may not actually agree with any of the items under review, and it is not therefore representative. An example of this will be seen when we take the case of four workmen, who earn respectively £4, £5, £6 and £7 per week. The average earnings of these four men are:—

$$\frac{£4 + £5 + £6 + £7}{4} = \frac{22}{4} = £5 \cdot 5$$

but not one of the workmen actually receives such a sum. Everyone, however, understands how this average is obtained and what is meant by it, and this alone has much to recommend its use.

(c) **Commercial Uses.**

The Arithmetic Average lends itself admirably to many types of Commercial Statistics, but care must nevertheless be taken to employ it properly. It has already been pointed out that in comparing any figures relating to various periods care must be taken to see that nothing abnormal occurs in either of the two under comparison at the moment. The Turnover of a business may be affected by a variety of causes. A "Boom" in trade will cause it to increase; a "Slump" will result in a decrease. Inflation of money, or, to put it the other way, a decrease in the purchasing power of money, will, *all other things being equal,* result in a leap upward in the Turnover, while an increase in the purchasing power of money, when other conditions remain unchanged, will, of course, bring about a fall. Increased competition and any change in the Revenue Duties payable upon a commodity (if any) will also affect it, and any one or more of these variations may happen in a very short period. To fix an arbitrary standard by which to measure anything does not allow for any changes of a permanent nature, or even of a semi-permanent nature, which may arise such as an increase in the Working Capital employed in the business, nor will it do to compare one year with another, since either or both may be subject to abnormalities which are incapable of being separated or assessed. If, however, we make a standard by averaging the results obtained from a number of years we shall have taken steps to eliminate to a very large extent many of the phenomena which would appear to be of a periodic nature, such as a cycle of trade. In choosing the length of time over which the average is to run care must be taken to see that such cycles of trade are usually covered, and it is, therefore, advisable to study the figures at one's disposal, in order to see that the period selected will in the normal way cover such cycle. If it is found that cycles of trade take five years, then a five years' average can be used; but if the cycles take ten years, then it is necessary to take a ten years' average. In ordinary trading concerns a ten years' average will be found to be the most useful. By taking the average over a large number of years we give bad years an opportunity of being offset by the good ones, or should no good ones be present, to nullify the effects of the bad ones, since then the effect of the poor periods is spread over a larger number of years. This would in effect be an Index Number, a subject which will be dealt with more fully later in this manual, and all future years would be compared by this Standard. Though this system has many advantages, it is not perhaps the most suitable for business use, for in business conditions change from year to year. The working capital of the concern may have been increased either through an additional issue of Shares or Debentures, or by leaving profits which have been realised in the business and so increasing the interest of the proprietors in the business. All other things being equal, an increase in the working capital at the disposal of a company or firm results in better opportunities for purchasing goods, and should logically increase the Turnover. Fluctuations are not therefore confined to outside disturbances, but may be caused by changing conditions within the business, and such changes should be provided for if our standard is to be a useful one.

### (d) Moving Average Standard.

It would therefore probably be better to make the standard a Moving Average one. This would be done by maintaining the same number of periods, but dropping off the earliest and taking in its place the last trading period, this procedure being repeated year after year. This would enable each year's Trading to be compared with the ten years' average (or such other average as may be adopted), ending with the previous trading period. The Standard thus obtained would have taken into account all fluctuations due to whatever cause right up to the beginning of the period under review.

### (1)—USE OF VARYING STANDARDS.

The effect of this may perhaps best be illustrated by an example. The following are the figures of the Turnover of a Company for the twenty years ended 1919. The Capital of the Company at the commencement of the period was £10,000, but Reserves amounting to £5,000 had been accumulated by 1914, and increased to £8,000 by 1919.

| | £ | | £ |
|---|---|---|---|
| 1900 .. .. | 35,000 | 1910 .. .. | 55,000 |
| 1901 .. .. | 42,000 | 1911 .. .. | 54,000 |
| 1902 .. .. | 43,500 | 1912 .. .. | 56,500 |
| 1903 .. .. | 47,000 | 1913 .. .. | 60,000 |
| 1904 .. .. | 48,500 | 1914 .. .. | 100,000 |
| 1905 .. .. | 52,000 | 1915 .. .. | 54,000 |
| 1906 .. .. | 36,500 | 1916 .. .. | 66,500 |
| 1907 .. .. | 37,500 | 1917 .. .. | 112,500 |
| 1908 .. .. | 53,000 | 1918 .. .. | 154,000 |
| 1909 .. .. | 53,500 | 1919 .. .. | 194,000 |

It will be observed that the Turnover fluctuated from year to year, due to many causes, the heavy fall in 1915 being probably attributable to the War, but so far the figures do not go far enough to enable us to decide how far the large Turnover in 1918 and 1919 will be maintained during post-war developments.

If we worked only by comparing the various years' Turnover with that of the first year, none of the changing conditions would be brought into consideration, for even though we were able to say by how much the Working Capital in, say, 1919 exceeded that of the Company in 1900, we cannot possibly decide by how much the turnover has increased directly as a consequence of the increased Working Capital. While basing our comparisons on the average of the ten years, 1900 to 1909 improves the Standard considerably, yet it does not take into account the changing conditions which applied during the second decade, particularly in the growth of the extra Working Capital in the shape of the Reserve. The moving average overcomes this objection, and in addition gives an indication whether the "Trend" of the business is upward or downward. Table F (see p. 54) shows the ratios which the Turnover of the years 1910 to 1919 bear to the bases obtained by each of the methods outlined above, the Ratios being calculated correct to the nearest whole number.

A comparison of columns (*a*), (*b*) and (*c*) is interesting, for while (*a*) shows a fall in 1915 as compared with the preceding years, the fall does not appear so considerable, while the jump upward in the following year is exaggerated owing to the low base used for the comparison. The same phenomena, but to a less extent, are also observable in column (*b*), but in column (*c*) the importance of the fall is emphasised as being *below* the standard of the trend of the business. It might be argued that 1914 was a better year due to increased Turnover following the known increase in the Working Capital, and that this fact would tend to exaggerate the fall in the following year; but this argument will not hold when we consider that the Reserves *had been* *accumulated and by* 1914 *had amounted to* £5,000, and further from column (*d*) we see that though the Turnover leapt upward very sharply in 1914, the effect upon the moving ten-year average was not extremely marked, for the average of the years 1904–1913 was £50,600, while for the years 1905–1914 (the basis of comparison with the year 1915) was only £55,750, though the Turnover had actually

### TABLE F.*

TABLE SHOWING THE RATIO TURNOVER BEARS TO STANDARDS CALCULATED ON VARYING BASES.

| Year. | (*a*) | (*b*) | (*c*) | (*d*) |
|---|---|---|---|---|
| | Base: Year 1900 =£35,000=100. | Base: Average of Ten Years, 1900–1909 =£44,850=100. | Base: Moving Ten-Year Average shown in Col. (*d*). | Ten-Year Moving Average for Col. (*c*). |
| 1910 | 157 | 123 | 123 | 1900–1909<br>£44,850 |
| 1911 | 154 | 120 | 113 | 1901–1910<br>£46,800 |
| 1912 | 161 | 126 | 118 | 1902–1911<br>£48,000 |
| 1913 | 171 | 134 | 122 | 1903–1912<br>£49,300 |
| 1914 | 286 | 223 | 198 | 1904–1913<br>£50,600 |
| 1915 | 154 | 120 | 97 | 1905–1914<br>£55,750 |
| 1916 | 190 | 148 | 119 | 1906–1915<br>£56,000 |
| 1917 | 317 | 251 | 190 | 1907–1916<br>£59,000 |
| 1918 | 440 | 343 | 232 | 1908–1917<br>£66,500 |
| 1919 | 554 | 433 | 253 | 1909–1918<br>£76,600 |

Basis for Comparison for 1920 = Years 1910–1919 = £90,650.

---

* The arrangement of this Table should be noted, as by placing the information in column (*c*) before that contained in column (*d*) we are enabled to compare the Ratios more easily than would be the case had the information in column (*d*) intervened between columns (*b*) and (*c*).

increased from £60,000 in 1913 to £100,000 in 1914. A further study of column (*d*) reveals the fact that the heavy fall in 1915 did not cause a very heavy sag in the moving ten-year average, for this average continued to advance throughout the ten years under review, and it is this moving ten-year average which shows the Trend of the business turnover.

### (2).—Short Term Fluctuations.

The average moving upwards would show that the trend of the Turnover was to increase steadily during the period, and that the falls which took place in the Turnover from time to time had no real effect on the progress of the business, so that by taking an average over a term of years we actually eliminate those fluctuations which, though having a marked effect at the moment, do not really affect the progress of the business at all. Such fluctuations are called "**Short Term Fluctuations.**" If data were available in relation to the particular trade or to trade in general, it would be possible to compare our ratios with those so published to see whether the business could show a better or a poorer result than the trade as a whole.

### (3)—Uses.

The principle which has been enunciated above may easily be employed in other directions. Instead of showing the percentage which Expenses bear to Turnover, which is at present the usual way of comparing the results of a current year with past periods, we could create for expenses a similar standard to that given for Turnover, and from this obtain the ratio which the Standard of expenses bears to the standard Turnover and compare such ratios with those of the current year. Thus variations would be shown and traced to their source, waste eliminated and excessive expenditure reduced. If the ratio of Turnover to the Standard advanced while the ratio of Current expenses to the Standard advanced by a similar or a less amount, it would appear that the management was pursuing a sound policy, and *vice versa*; hence, if the standard for expenses was 50 to 100 for Turnover, and in the current year was 70 for expenses and still 100 for Turnover, inquiries would of necessity be made, for expenses must have risen considerably in the period under consideration, and the causes of the rise should be traced to their source, as it may be found that it is due to some condition which is capable of being remedied, or some inefficiency which should be corrected. To carry out such an inquiry properly it may be necessary to analyse the items composing expenses and compare them with the corresponding items in the standard of expenditure, following the same procedure in each case. With expenses it is essential that the moving Standard be adopted, for many of the component items will fluctuate considerably from year to year, particularly such items as Wages, Salaries Rates and Taxes.

## § 3.—The Weighted Arithmetic Average.

Though the simple Arithmetic Average can be used in many kinds of Statistical work, it has its limitations, and these are best illustrated by an example.

The employees of a firm number 10,000, and are divided into three groups as follows:—

(1) 4,000 Skilled workers with an average weekly wage of £5.

(2) 3,500 Semi-skilled workers with an average weekly wage of £4.

(3) 2,500 Unskilled workers with an average weekly wage of £3.

It is desired to ascertain the average wage received per worker. This might, at first glance, appear to be:— $\frac{£5 + £4 + £3}{3}$ or £4, and if this were so, the weekly wage bill of the firm would be £4 × 10,000 = £40,000; but, as a matter of fact, the actual pay roll amounts to:--

$$(\pounds 5 \times 4{,}000) + (\pounds 4 \times 3{,}500) + (\pounds 3 \times 2{,}500)$$
$$= \pounds 20{,}000 + \pounds 14{,}000 + \pounds 7{,}500 = \mathbf{\pounds 41{,}500}$$

so that our average is clearly incorrect. But if we multiply the average wages of the men in each group by the number of the men in each group, and add the results, and then divide by the total number of workers, we get :—

$$\frac{(\pounds 5 \times 4{,}000) + (\pounds 4 \times 3{,}500) + (\pounds 3 \times 2{,}500)}{10{,}000}$$
$$= \frac{\pounds 20{,}000 + \pounds 14{,}000 + \pounds 7{,}500}{10{,}000} = \frac{\pounds 41{,}500}{10{,}000} = \mathbf{\pounds 4 \cdot 15}$$
$$= \mathbf{\pounds 4\ 3s.}$$

which is the actual average of the whole, since

$$\pounds 4\ 15\text{s.} \times 10{,}000 = \pounds 41{,}500,$$

the amount of the weekly Wage bill.

### (a) Definition.

This type of Average is known as the "**Weighted Arithmetic Average,**" and may be described as *an average whose component items have been multiplied by certain numbers known as "Weights" before being added, the aggregate being then divided by the sum of the weights, instead of by the number of items.* If the numbers of men in each of the above groups of workers had been the same, then weights need not have been used, as the divisions being equal would have caused a Simple Arithmetic Average to suffice. This principle of the Weighted Average is very important in all cases where varying quantities are in evidence, and it will be necessary, for instance, to use it in a factory where average cost per unit of the commodities manufactured is desired, and the output varies for the various commodities, also where the average output per machine is required, and the machines are of different patterns or are working under varying conditions.

**(b) The Weights to be used.**

It is not, however, necessary to use the exact weights. They may be estimated, but if the number of weights used is small, the size of those used may have a marked effect upon the result, and therefore the weights used should be approximately correct. It has been demonstrated by Professor Bowley in his Book *The Elements of Statistics* that mathematically an *error in the Weights tends to be much less serious than a corresponding error in the size of the items,* for an error in the weights, if many of them are used, will in all probability be Unbiassed, and so tend to be corrected, The following example will suffice to illustrate this fact. Three countries have an acreage of 3,542,160, 2,673,940 and 1,340,920 respectively under Corn crops, and produce an average of 35·4, 29·6 and 31·2 bushels per acre. What is the average yield per acre of the ground under cultivation ?

(*a*) Using actual figures we get :—

$$\frac{(35\cdot4\times3{,}542{,}160)+(29\cdot6\times2{,}673{,}940)+(31\cdot2\times1{,}340{,}920)}{3{,}542{,}160+2{,}673{,}940+1{,}340{,}920}$$

$$=\frac{125{,}392{,}464+79{,}148{,}624+41{,}836{,}704}{3{,}542{,}160+2{,}673{,}940+1{,}340{,}920}=\frac{246{,}377{,}792}{7{,}557{,}020}$$

$$=\mathbf{32\cdot6025}$$

(*b*) If we now approximate the figures for the acreage to the millions correct to the first decimal place we get :—

$$\frac{(35\cdot4\times3\cdot6)+(29\cdot6\times2\cdot7)+(31\cdot2\times1\cdot3)}{3\cdot6+2\cdot7+1\cdot3}$$

$$=\frac{127\cdot44+79\cdot92+40\cdot56}{3\cdot6+2\cdot7+1\cdot3}=\frac{247\cdot92}{7\cdot6}=\mathbf{32\cdot62105}$$

a difference between the result obtained in (*a*) of only $+ \cdot 01855$ of a bushel.

(*c*) If, however, we approximate the average yields to the nearest bushel and work with the correct acreage the result is as follows :—

$$\frac{(35 \times 3{,}542{,}160) + (30 \times 2{,}673{,}940) + (31 \times 1{,}340{,}920)}{3{,}542{,}160 + 2{,}673{,}940 + 1{,}340{,}920}$$

$$= \frac{123{,}975{,}600 + 80{,}218{,}200 + 41{,}568{,}520}{3{,}542{,}160 + 2{,}673{,}940 + 1{,}340{,}920} = \frac{245{,}762{,}320}{7{,}557{,}020}$$

$$= \mathbf{35 \cdot 52106}$$

a difference from the result obtained in (*a*) of $- \cdot 08144$ of a bushel, a much larger error than is found in (*b*), and one which in this case would tend to grow as the numbers were increased.

### (c) When Weighted Average must be used.

Unless weights are used, observations and deductions are apt to be fallacious. Generally speaking, unskilled labour is predominant in the world, and wages for this class of worker are lower than in the skilled branches of industry, consequently, if the simple arithmetic average alone were used, a highly paid foreman would rank as equal to a number of men, and his higher scale of remuneration would make the average wage earned much higher than it is in reality, but the use of weights would counteract this, as the following example will show :—

*A Works employs three foremen at a wage of £20 per week each, and 120 workmen whose average wage is £1 per week. What is the average wage?*

If we use the Simple Arithmetic Average, we get :—

$$\frac{£20 + £1}{2} = \frac{21}{2} = £10 \cdot 5$$

a result which is obviously misleading. If, however, we use weights, we get the following result :—

$$\frac{(£20 \times 3) + (£1 \times 120)}{3 + 120} = \frac{60 + 120}{3 + 120} = \frac{180}{123} = \mathbf{£1{\cdot}463}$$

a result which is clearly more correct than the first.

### (d) Effect of Changing Proportions.

In the same way, if Wages are increased and the numbers of the workers in the different classes also change, the change in the average would be much smaller than would appear possible. If we again refer to the case above of 10,000 workmen and assume that all the workers, irrespective of the grade to which they belong, received an advance of £1 per week, the average wage would then become :—

$$\frac{(£6 \times 4{,}000) + (£5 \times 3{,}500) + (£4 \times 2{,}500)}{4{,}000 + 3{,}500 + 2{,}500}$$

$$= \frac{24{,}000 + 17{,}500 + 10{,}000}{4{,}000 + 3{,}500 + 2{,}500} = \frac{51{,}500}{10{,}000} = \mathbf{£5{\cdot}15}$$

If at the same time as the change in wages was made the number of skilled men employed was reduced by 1,000, the semi-skilled by 500, while the Unskilled Workers were increased by 1,500, the number of workers would be unchanged, but the average wage would now be :—

$$\frac{(£6 \times 3{,}000) + (£5 \times 3{,}000) + (£4 \times 4{,}000)}{3{,}000 + 3{,}000 + 4{,}000}$$

$$= \frac{18{,}000 + 15{,}000 + 16{,}000}{3{,}000 + 3{,}000 + 4{,}000} = \frac{49{,}000}{10{,}000} = \mathbf{£4{\cdot}9}$$

an average increase over the average prior to the change of $4{\cdot}9 - 4{\cdot}15 = £0{\cdot}75$, or only 15s. per week. If the number of men in any group changed, or the proportion in each group alters, the weighted average will change also.

**(e) Commercial Uses.**

The " Moving Average " can, of course, be applied to the Weighted Average with equal facility and usefulness as to the Simple Arithmetic Average, and it is frequently used by merchants to ensure that their dealings will yield them an ultimate profit, though individual transactions may apparently show a loss. The following series of transactions will illustrate the method :—

TABLE G.

TABLE ILLUSTRATING THE APPLICATION OF THE MOVING AVERAGE.

| Date. | No. of Bales Purchased. | Price per Bale. | Total Bales Purchased. | Amount Paid. | Total Amount Paid. | Average Price per Bale. |
|---|---|---|---|---|---|---|
| | | £ | | £ | £ | £ |
| January 1 | 100 | 2·0 | 100 | 200 | 200 | 2·0 |
| | 100 | 1·8 | 200 | 180 | 380 | 1·9 |
| | 50 | 1·4 | 250 | 70 | 450 | 1·8 |
| January 2 | 250 | 1·6 | 500 | 400 | 850 | 1·7 |
| | 100 | 2·3 | 600 | 230 | 1,080 | 1·8 |
| January 3 | 400 | 1·3 | 1,000 | 520 | 1,600 | 1·6 |

As each purchase is made a Weighted Average is extended showing the average cost per unit of the goods purchased, and from this moving average the merchant can calculate the price at which he can sell the goods or part of them to ensure a profit. In the foregoing example, after the merchant had made his first purchase, he could not sell any of the goods at under £2 per bale unless he made a loss, but after the second purchase had been made he could afford to sell the whole of his 200 bales at £2 per bale and then make a profit, while his third purchase brought the average cost per bale down to £1·8, and therefore a sale made at anything over this figure would yield him a profit. This system would have to be adopted for every commodity separately, and is particularly suitable when dealing on a fluctuating market. The same procedure is also frequently adopted on the Stock Exchange by Speculators who desire to snatch a profit where a loss seems imminent. Supposing a purchase is made of 1,000 shares at 30s. per share in anticipation of the price rising and being able to sell at a profit, but the anticipation is not realised, and instead of advancing the price fell steadily, thus not only is a profit apparently impossible, but a loss seems inevitable. Let us assume that the shares fell to 25s., at which price a further purchase of 2,000 shares is made. The average cost per share of the 3,000 shares would now be :—

$$\frac{(\pounds 1\cdot 5 \times 1{,}000) + (\pounds 1\cdot 25 \times 2{,}000)}{3{,}000} = \frac{1{,}500 + 2{,}500}{3{,}000}$$
$$= \pounds 1\ 6s.\ 8d.$$

so that if the price subsequently advanced to 29s. per share, they could be sold to yield a profit of 2s. 4d. per share, or a total profit on the whole of the transactions of 2s. 4d. × 3,000 = £350, although the Selling Price did not reach the figure at which the first 1,000 Shares were bought.

Such a Moving Weighted Average would enable many a Trader to increase his business by adjusting his prices to market conditions. It might be argued that the compilation of such records takes time, but on the other hand most business men use the system in some form or another, and the only additional thing therefore is to do it systematically, instead of, as is the general rule now, when the market is moving rapidly against the operator. If it were always available, transactions might be carried out successfully in the face of severe competition, when competitors had not the same data available.

## § 4.—Descriptive Average.

If the data on which we are working are not exact, or are incomplete, it follows that the averages compiled from such data will contain an unknown and unmeasureable error. This happens when 100 Workers are described as each earning between £3 10s. and £4 per week, and we assume the average wage to be £3 15s. which may be entirely incorrect, as we have estimated the numbers receiving varying rates of pay as being equal, whereas they may vary considerably. Such an average may be described as a "**Descriptive Average,**" and it is then known that it cannot be relied upon for further working. It, however, sometimes becomes advisable to obtain a descriptive average as a guide—but as a guide only—to varying conditions, and the method of obtaining it is illustrated in the following example :—

What is the "average" wage of 75 workers, of whom 10 earn 10s. or less per week, 20 earn over 10s. and not over 20s., 30 earn wages exceeding 20s. and up to 30s., and the remaining 15 earn over 30s., but not over 40s. ?

It is necessary to assume that the wages are evenly distributed over the groups (*i.e.* in the first group the first worker receives 1s., the second 2s. and so on), and the average rate received by the men in the first group of ten men would therefore be :—

$\frac{1+2+\ldots+10}{10} = 5\cdot5$s., and we assume that each group receives the same average plus the number of units of the difference of the wages between the groups (in this case 10s.), by which the wages of the succeeding groups exceeds the first group. When this is done we get :—

| Group No. | (b). | (b). | (c). | (d). | (e). |
|---|---|---|---|---|---|
| | Rate of Wages. | No. of Men in Group. | Wages. | Wages in Units of 10s. | Product of Columns (b) and (d). |
| | | | s. | | |
| 1 | 10s. and less .. | 10 | (say) 5·5 | 5·5 + 0 | 0 |
| 2 | Over 10s. and not exceeding 20s. .. | 20 | 5·5 + 10 | + 1 | 20 |
| 3 | Over 20s. and not exceeding 30s. .. | 30 | 5·5 + 20 | + 2 | 60 |
| 4 | Over 30s. and not exceeding 40s. .. | 15 | 5·5 + 30 | + 3 | 45 |
| | | 75 | | | 125 |

Columns (*b*) and (*e*) are then totalled, and the following is the formula to find the average :—

$$\text{Assumed average of first group} + \left(\frac{\text{Total of column } (e)}{\text{Total of column } (b)} \text{ of the unit of increase}\right).$$

so that in the above case we get :—

$$5\cdot5 + \left(\frac{125}{75} \text{ of } 10\text{s.}\right) = 5\cdot5\text{s} + 16\cdot6\text{s.} = \mathbf{22s.\ 3d.}$$

If the numbers in column (*b*) are large, then we can use approximate numbers in place of the actual numbers without materially affecting the result. Thus in the example given above the numbers in column (*b*) might have been approximated to 1, 2, 3 and 1·5 respectively.

## § 5.—Median.

### (a) Arraying the Data.

If a number of similar objects are placed side by side in order of their magnitude (*i.e.* the smallest on the left and the largest on the right, or *vice versa*), they are said to be

"**Arrayed,**" and the middle example of the array is known as the "**Median,**" and may be said to be that item whose magnitude most closely corresponds to the magnitude of all the other examples in the array when such examples are numerous. Diagram No. 1 (see p. 95) shows eighty items, ranging in size from 1 to 70, arrayed, and the sizes may represent any data desired. The centre item (*i.e.* the fortieth) is the Median. In such a diagram the size of any item is easily ascertained by reference to the vertical scale. The foregoing example is a very simple form, but frequently our information is not so definite, and our examples have to be grouped as falling within certain limits, so that to plot these accurately is impossible, as the size of the items in the different groups is only known to be within certain limits, and hence the magnitude of the Median cannot be accurately determined.

**(b) Frequency Tables.**

This is illustrated in the following case. There is a group of 261 workers, and of these 50 earn not more than 10s. per week, 70 earn over 10s. but not exceeding 12s., 60 earn from 12s. to 14s., and 81 take from 14s. to 16s., and it is desired to ascertain what is the probable wage of the Median example. We first compile what is known as a "Frequency Table," which is constructed as follows:—

| Group No. | (*a*) Amount of Wages.* | (*b*) Number of Workers receiving such Rate of Wages.† | (*c*) Cumulative Numbers. |
|---|---|---|---|
| 1 | Not exceeding 10s. | 50 | 50 |
| 2 | Over 10s., but not exceeding 12s. | 70 | 120 |
| 3 | Over 12s., but not exceeding 14s. | 60 | 180 |
| 4 | Over 14s., but not exceeding 16s. | 81 | 261 |

---

* Column (*a*) may be described as the "Size" of the data under consideration.

† Column (*b*) gives the "Frequency," *i.e.* the number of the particular size under review.

From the data given in columns (*a*) and (*b*) we cannot obtain the Median by inspection, and hence it is necessary to re-group the classes by taking the Cumulative Frequency as shown in column (*c*), from which we obtain the information that there are 120 workers earning varying sums of not more than 12s. per week, 180 take anything not exceeding 14s. per week, and so on. The Median is the 131st case, and will now be seen to be in group No. 3, *i.e.* the group in which the workers earn varying sums from 12s. to 14s. per week, and is the eleventh man of that group. We therefore know that the particular individual earns between 12s. and 14s. per week, and if it be desired to ascertain what is the approximate amount of his earnings it is necessary to assume that the "CLASS INTERVAL" (*i.e.* the difference between the highest wages in the class below the one in question and the lowest of the class above) is spread evenly over the workers in the group. This is known as "**Interpolation.**"

### (c) Rule for Calculating the Magnitude of the Median.

The earnings of the Median man may now be estimated to be :—

The highest amount received in the group below

$$+\left(\frac{\text{Median number—Total number of cases to end of class below}}{\text{Number of men in the class in which the Median is situated}}\quad\text{of the class interval}\right)$$

Apply this to the data we get :—

$$12\text{s.}+\left(\frac{131-120}{60}\text{ of 2s.}\right)=12\text{s.}+\left(\frac{11}{60}\text{ of 2s.}\right)$$

$$=12\text{s.}+4\cdot4\text{d.}=\mathbf{12s.\ 4\cdot4d.}$$

A simpler formula than that given above would be :—
The highest amount received in the group below

$$+\left(\frac{\text{Position of Median in group containing it}}{\text{Number of men in group containing the Median}}\ \text{of the class interval}\right)$$

$$i.e.\ 12\text{s.} + \left(\frac{11}{60}\ \text{of } 2\text{s.}\right)$$

which gives the same result as above.

**(d) Quartiles.**

If we decide to divide the array further we can take the items placed midway between the Median and the extremes, *i.e.* we divide the array into four parts. These units are known as "**Quartiles.**" In the Diagram No. 1 the items 20 and 60 will be the Quartiles, the former, which is between the lower extreme and the Median, is known as the "**Lower Quartile,**" while the latter being between the Median and the upper extreme is called the "**Upper Quartile.**" Like the Median, the Quartiles are merely representative. They are easily ascertained from a graph, and their position can usually be ascertained by calculation; but when our data is in the form of a frequency table it is better to locate them in the same manner as the Median. In the foregoing Frequency Table the Lower Quartile is the 66th case, and is the sixteenth item in the second group, which contains seventy cases. Its magnitude therefore is :—

$$10\text{s.} + \left(\frac{16}{70}\ \text{of } 2\text{s.}\right) = 10\text{s.} + 5{\cdot}48\text{d.} = \mathbf{10s.\ 5{\cdot}48d.}$$

**(e) Deciles and Percentiles.**

An array may also be divided into Tenths, in which case the tenth items are known as "*Deciles,*" or into hundredths, in which case the representative items are called "*Percentiles.*"

**(f) Advantages and Disadvantages.**

The Median has the advantage of being capable of exact location, but to counteract this it has the disadvantage of being incapable of measurement by any simple mathematical formulæ. Unlike the Arithmetic Average the value of the total of the items cannot be obtained even after we have obtained the value of the Median, since it is one example only, and the remaining items of the array may vary widely. The Median is not affected to any great degree by the deviations of the extremes, and moreover it is not necessary to know the size of the items at the extremities of the data in order to ascertain the Median, provided we know the exact number of the items. Thus, to return for a moment to our community of artisans and millionaires it would be unnecessary for us to know what the income of the millionaires actually was if we were to obtain the Median and use this as the representative income of the community, but it would be necessary to obtain this information before the correct Arithmetic Average could be obtained. The Median possesses a valuable advantage over the Arithmetic Average in so far that it is an actual case in the data, and so in a way is more representative than the Arithmetic Average, and from this it will appear clear that it is very useful for considering data which it is impossible or difficult to group into definite classes. It is particularly useful when considering social phenomena or conditions such as Wages, Distribution of Wealth, Ability, etc., but from a Commercial point of view will probably be considered too indefinite and too isolated to be of great use.

## § 6.—Mode.

If we again refer to Diagram No. 1, it will be seen that of the eighty items there arrayed the majority vary in size from 1 to 70, but that a series of ten items (*i.e.* items 40 to 49) are uniform in magnitude. If therefore we were to shuffle this array of data, and pick out an item at random, it is obvious that the one most likely to be chosen would be one of the size 40, since there are ten of these against seventy of varying sizes. It will be found in studying phenomena that generally speaking there will be items which are more numerous than others. Such predominant items are known in Statistical Science as the "**Mode.**" When the "Average Man" is spoken of we mean the type of man most frequently met with, and it would therefore be better in many ways to refer to him as the "Modal" man, rather than as the Average Man.

### (a) Method of Locating the Mode.

On a chart showing the array such as is the case in Diagram No. 1 the Mode is easily distinguishable, but when the examples of each size are at all numerous, and the sizes themselves are nearly uniform, it is more difficult to locate, and it is frequently necessary to regroup the items into other groups and continue this process till one group stands out pre-eminent, so as to enable the Modal group to be identified, though even then the Mode itself can only be approximately identified, though the limits of error are usually very small.

The Table appearing on p. 70 shows the method of regrouping. Column (*a*) shows the size of the items, and (*b*) the frequency, or the number of items of each particular size.

It will be seen from Column (*b*) that no item has such a frequency that it stands out pre-eminently, though the item of size 18 with 51 examples would appear for the moment to be the Mode, but as the frequencies of many of the items are so similar it is necessary to ascertain more definitely if our first inspection is correct. We shall therefore group the items into fresh classes with a view to ascertaining if any other factor is likely to show a great predominance. Column (*c*) shows the number of examples in the new groups, *e.g.* it will be seen that there are seventy-two examples in the group composed of those items of sizes 10 and 11, and so forth. The process of grouping the items in pairs is continued in column (*d*), but sizes 11 and 12, 13 and 14, and so on would be combined, thus each of the original classes is combined with that before and after it. In column (*c*) the Mode would appear to be in that group which contains items of the sizes of 18 and 19, as there are 100 examples in this new group, but in column (*d*) the largest group is seen to be that of sizes 15 and 16, and as the location is still no clearer it is necessary to regroup the data. This time we combine three

| Size. | Frequency. | | | | | |
|---|---|---|---|---|---|---|
| (*a*) | (*b*) | (*c*) | (*d*) | (*e*) | (*f*) | (*g*) |
| 10 | 34 | 72 | | 114 | | |
| 11 | 38 | | 80 | | 126 | |
| 12 | 42 | 88 | | | | 136 |
| 13 | 46 | | 94 | 144 | | |
| 14 | 48 | 98 | | | 148 | |
| 15 | 50 | | 100 | | | 143 |
| 16 | 50 | 93 | | 144 | | |
| 17 | 43 | | 94 | | 143 | |
| 18 | 51 | 100 | | | | 142 |
| 19 | 49 | | 91 | 129 | | |
| 20 | 42 | 80 | | | 116 | |
| 21 | 38 | | 74 | | | |
| 22 | 36 | | | | | |

of the original classes starting with the first three. The result is seen in column (*e*), and here we obtain two groups with the same number of examples, *i.e.* 144 in each case. Column (*f*) is then prepared again grouping in classes of three, and this is done by shifting the limit of the group one class down, and the process is repeated in constructing column (*g*). When we group two classes together we do so twice, moving the upper limit of the group down one class the second time, and when grouping three classes we repeat the process twice, proceeding in a similar manner by shifting the upper limit down one each time. If no group is now predominant it will be necessary to group four classes together four times, and so on till we get a predominant group, though it must be borne in mind that each time we find it necessary to bring another class into the new group we are making the location of the Mode more approximate. In the example under review we find that columns (*f*) and (*g*) show groups which are distinctly larger than the others, and, moreover, the largest group seems to centre on the same spot. An analysis will show which class will appear constantly in the maximum group. Thus in column (*f*) we find sizes 14, 15 and 16 in the maximum group. In column (*g*) the sizes are 15, 16 and 17. It will thus be seen that the sizes 15 and 16 appear in each of the maximum groups, and that no other example does so, and we are therefore able to say that the Modal size is approximately between 15 and 16, and this result is borne out if we carry our grouping further.

### (b) Advantages and Disadvantages.

A study of Diagram No. 1 at once shows that the Mode is fixed only by the predominant examples, and therefore that the extremes, however large they may be, do not affect it at all. Like the Median, therefore, it has an advantage in this respect over the Arithmetic Average. To return once more to our community of artisans and millionaires, no question could arise as to the average income of the community being affected by the income of the millionaires if the Mode were taken instead of the Simple Arithmetic Average. Like the Median also, but unlike the Arithmetic Average, the Mode represents an actual case, but it is even better than the Median in this respect, for whereas the Median will be a single example, the *Mode is that example which is the most likely to occur.* While the Median may be affected to some slight extent by the extremes, and it is necessary to know what they are if the Median is to be correct, the Mode can be ascertained without that information provided it is known that the extremes are small in number as compared with the Modal group. The Mode, however, suffers from the disadvantage that a comparatively small number of items of the same size in a large number of varying sizes will determine it. The Mode is frequently difficult to determine, and cannot be located by Arithmetical processes, and, moreover, it is useless if we wish to obtain a correct aggregate.

### (c) Commercial Uses.

The Mode has so far not been used to any great extent in Commercial Statistics, but since it is the case which is most likely to be met with it should prove a reliable guide to what will happen when similar data are available, or work is to be done. If, for instance, we record the Output of a number of similar machines engaged on the same class of work, we shall find that the Output per machine varies, owing either to the labour engaged varying in skill, or because one machine is not so favourably situated; but without doubt it will be found that a number of machines will give practically the same output, and the Modal Output might thus be determined. Knowing the Output given by a number of machines, inquiries could then be made as to the reasons why the others varied from this Mode, and thus lead to greater uniformity of Output and consequently greater efficiency. It would also be possible to estimate more correctly the length of time necessary to secure a certain output. In the same way it will be found that if Time Records be made of work done, a Modal Time will probably stand out, and will represent the most likely time to be taken to execute a similar piece of work, and hence assist materially in forming the basis for a Time Contract. They are admittedly only possibilities, but they are possibilities founded upon an exact knowledge of past events, and are thus more likely to be correct than any other method of estimating.

## § 7.—Geometric Average.

There is also the "**Geometric Mean,**" or Average, which is found in the following manner: If the number of items in a series $= n$, we multiply the items together and extract the $n$th root of the product. It is difficult of calculation, and often involves much work even when the aid of Logarithms is invoked, and consequently is little used in Statistical Work. This average, however, gives very little weight to extreme variations, and in this respect is between the Median and the Arithmetic Average. The Geometric Mean is always lower than the Arithmetic Average. Its use may be shown thus:—the average rate of increase in an item when three successive periodical increments (say, monthly, quarterly or yearly) are 40, 50 and 60 per cent.

is not $\dfrac{40 + 50 + 60}{3} = 50$ per cent., but

$$= \sqrt[3]{(1{\cdot}40 \times 1{\cdot}50 \times 1{\cdot}60)} \times 100 - 100$$

$$= \sqrt[3]{3{\cdot}36} \times 100 - 100 = 1{\cdot}497 \times 100 - 100 = 49{\cdot}7 \text{ per cent.}$$

Owing to the difficulties of calculation it will not be used in Commercial work to any extent, and can therefore be omitted from further consideration.

# *PART II.*

# PRESENTATION AND USES OF STATISTICAL DATA.

## SYNOPSIS TO CHAPTER V.

## TABULATION.

§ 1.—Importance of Proper Tabulation.

2.—Kinds of Tabulation.
  (*a*) Simple.
  (*b*) Complex.

3.—Rules for Tabulation.

4.—Data to be Tabulated.

5.—Importance of Checking Calculations.

6.—Title of the Table.

7.—Tabulation of Commercial Data.

## CHAPTER V.

# TABULATION.

However carefully our records are compiled, it is essential that they be presented to those for whom they are intended in such a manner as will enable them to make the best use of the information given, and, moreover, so that they will at once realise the importance of significant factors which appear in the data under review. The method of presentment is important, since a good cause may be irreparably damaged if the case be not correctly placed before the people interested, while a poor case may win support if the facts are carefully, logically and succinctly drawn up so as to excite the interest of the reader. It has been said that the facts to be presented do not matter so much as the method of presenting them, and this statement is in some measure true, for the speed of the World's commerce to-day is such that business opportunities with their resultant profit may be lost if time has to be spent in an exhaustive study of data which may or may not assist the business man in coming to a decision. The important points of a business must be available in such a form that they can be speedily obtained, easily understood, and their significance grasped without undue study and waste of time. Facts must therefore be presented in such a way that they attract the attention, give the information required, and enable the results following on a definite action to be foreseen and provided for. The Presentment of Data has now reached a high standard in some of our Newspapers, the headlines of which supply such information that the busy man is enabled to grasp the salient features of the news without the necessity of voluminous reading of details which frequently are of minor importance. Time is thus saved for concentration on the affairs which are of more vital interest.

## § I.—Importance of Proper Tabulation.

In Statistical work we rely upon two forms of presentation, viz.: Tabulation of Results and Graphic Presentment, and both present some difficulties in their preparation. Even when the Graphic method is used Tabulation is necessary, though sometimes to only a limited extent, before the form our diagram is to take can be decided upon. The Tabulation of results is really the final stage of the work of compilation, and subsequently becomes the basis for consideration, deduction and application to present and future problems. It consists of the classification of the data obtained by our inquiries, sampling, recording and averaging. In preparing our Tables we must carefully consider the reason for which the data are required or have been assembled, and also the ultimate uses to which they will be put, and we can then decide the form the Tabulation shall take in order that the information may be shown clearly, succinctly and yet briefly. Tabulations must be capable of being easily read, and must necessarily, therefore, be compact in form, and should be so arranged that those figures which have to be compared will be in close proximity to one another. This has already been demonstrated in Table G, where the information contained in column (*c*) was placed before that in column (*d*), although the latter data had to be obtained before the details given in the former column could be calculated. A small Table enables all these points to be dealt with easily, but frequently our data are of such magnitude that further divisions become necessary, and when this is the case the Table tends to become unwieldly, and its value is decreased in a direct ratio. Generally speaking, each Table should be definitely prepared to show the particular point it is desired to demonstrate, for planning a tabular statement to enable comparison to be made between several distinct and differing units is apt to lead to confusion to those not well acquainted with the particular details under review. Care must also be taken to see that the units which are being compared are such as will enable comparison to be made. It would, for instance, serve no very useful purpose to show in the same statement with the object of comparison the *Output* of a business and the *Gross Profit,* since Gross Profit bears no direct relationship to the quantity of goods produced; but, on the other hand, *Sales* (or Turnover) can be compared with *Gross Profit,* since the latter depends to a large measure upon the former, though it must be borne in mind that profits will not of necessity fluctuate to the same degree as the Turnover, for even if the price of the commodity rises, but expenses change in a different ratio, the profit will not increase to the same extent as the Turnover.

## § 2.—Kinds of Tabulation.

We can prepare our Tabular Statement in either a **simple** or a **complex** form.

### (a) Simple.

**Simple Tabulation** is designed to supply information in respect of one or more groups of independent inquiries. The following Table is an example of Simple Tabulation :—

TABLE H.*

NUMBER AND TOTAL AMOUNT OF NOMINAL CAPITAL OF COMPANIES REGISTERED IN THE UNITED KINGDOM UNDER "THE COMPANIES (CONSOLIDATION) ACT, 1908," IN EACH OF THE UNDERMENTIONED YEARS.

| Years. | Total Number of Companies. | Total Nominal Share Capital. |
|---|---|---|
| | | £ |
| 1909 | 6,373 | 141,630,296 |
| 1910 | 7,184 | 212,975,689 |
| 1911 | 6,444 | 157,303,062 |
| 1912 | 7,367 | 174,004,837 |
| 1913 | 7,425 | 157,186,653 |
| 1914 | 6,214 | 113,251,583 |
| 1915 | 4,062 | 53,354,606 |
| 1916 | 3,393 | 50,442,871 |
| 1917 | 3,963 | 67,813,926 |

**(b) Complex.**

**Complex Tabulation** shows the division of a Total into two or more categories, as in Table J (opposite), where the total number of Companies and their Paid-Up Capital which are registered in the United Kingdom are divided into those registered in England, Scotland and Ireland respectively.

Another form of Complex Table is one where the separate columns can be totalled, and the grand total checked by the cross-casts of the columnar table, of which Table K (p. 80) is an example.

## § 3.—Rules for Tabulation.

Extra columns may be added or additional information tabulated at the discretion of the compiler, and it must be remembered that the more minute the divisions the greater the accuracy of the Table becomes; but, on the other hand, every additional column makes the Table more difficult to read, causes comparison to be less effective, and is likely to lead to confusion of thought and possible error, all of

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* Compiled from the "Statistical Abstract for the United Kingdom."

## TABLE J.*

TOTAL NUMBER AND AMOUNT OF PAID-UP CAPITAL (INCLUDING AMOUNTS CONSIDERED AS PAID ON VENDORS' AND OTHER SHARES) OF ALL REGISTERED COMPANIES HAVING A SHARE CAPITAL, AND BELIEVED TO BE CARRYING ON BUSINESS ON THE 30TH APRIL OF THE UNDERMENTIONED YEARS.

| On 30th April of the Years. | Registered in England. | | Registered in Scotland. | | Registered in Ireland. | | Total. | |
|---|---|---|---|---|---|---|---|---|
| | No. | Paid-up Capital. | No. | Paid-up Capital. | No. | Paid-up Capital. | No. | Paid-up Capital. |
| | | £ | | £ | | £ | | £ |
| 1909 | 41,257 | 1,937,196,022 | 3,659 | 180,889,337 | 1,558 | 45,047,432 | 46,474 | 2,163,132,789 |
| 1910 | 46,301 | 1,949,328,412 | 3,836 | 183,307,207 | 1,650 | 45,984,115 | 51,787 | 2,178,619,734 |
| 1911 | 47,996 | 1,986,037,231 | 3,993 | 189,783,540 | 1,718 | 46,473,203 | 53,707 | 2,222,293,974 |
| 1912 | 50,425 | 2,093,460,607 | 4,134 | 194,525,269 | 1,793 | 47,217,965 | 56,352 | 2,335,203,841 |
| 1913 | 54,535 | 2,178,244,815 | 4,347 | 198,722,237 | 1,872 | 48,773,805 | 60,754 | 2,425,740,857 |
| 1914 | 58,164 | 2,276,596,067 | 4,598 | 205,273,585 | 1,930 | 50,078,009 | 64,692 | 2,531,947,661 |
| 1915 | 59,287 | 2,395,976,218 | 4,682 | 210,396,181 | 2,017 | 51,094,565 | 65,986 | 2,657,466,964 |
| 1916 | 59,322 | 2,449,282,026 | 4,716 | 215,929,381 | 2,056 | 51,777,722 | 66,094 | 2,716,989,129 |
| 1917 | 59,254 | 2,468,812,058 | 4,756 | 216,458,249 | 2,121 | 52,462,827 | 66,133 | 2,737,733,134 |

* Compiled from the "Statistical Abstract for the United Kingdom."

## TABLE K.*

### MILEAGE RUN BY RAILWAY COMPANIES' ENGINES DURING FOUR WEEKS ENDED 29TH FEBRUARY, 1920.

| Railway Company. | Train Miles (including Empty Trains run for Traffic Purposes). | | Shunting Miles. | | "Other Miles" (excluding Departmental). | | Departmental Miles. | Total. |
|---|---|---|---|---|---|---|---|---|
| | Coaching. | Freight. | Coaching. | Freight. | Coaching. | Freight. | | |
| ENGLAND AND WALES: | | | | | | | | |
| Great Central .. | 619,626 | 686,385 | 79,294 | 721,104 | 51,880 | 178,814 | 44,839 | 2,381,942 |
| Great Eastern .. | 1,025,065 | 635,594 | 63,726 | 381,030 | 45,959 | 45,900 | 104,789 | 2,302,063 |
| Great Northern .. | 678,019 | 663,140 | 48,049 | 388,570 | 46,281 | 79,614 | 87,542 | 1,991,215 |
| Great Western .. | 1,878,758 | 1,645,489 | 147,339 | 1,389,597 | 84,140 | 298,440 | 132,291 | 5,576,054 |
| L. & Y. .. .. | 844,224 | 407,023 | 95,315 | 732,537 | 48,662 | 170,662 | 94,349 | 2,392,772 |
| L. & N.W. .. .. | 1,837,917 | 1,489,905 | 232,780 | 1,125,698 | 170,405 | 294,267 | 173,481 | 5,324,453 |
| L. & S.W. .. .. | 1,104,492 | 275,346 | 75,987 | 280,515 | 56,243 | 62,603 | 41,739 | 1,896,925 |
| L.B. & S.C. .. | 701,540 | 142,537 | 34,196 | 122,893 | 34,357 | 20,948 | 28,187 | 1,084,658 |
| Midland .. .. | 1,301,061 | 1,890,927 | 102,583 | 1,089,068 | 99,489 | 516,431 | 171,643 | 5,171,202 |
| North Eastern .. | 1,007,467 | 1,037,191 | 71,637 | 977,441 | 46,794 | 129,896 | 108,673 | 3,379,099 |
| North Staffs .. | 115,675 | 88,564 | 14,315 | 117,590 | 1,412 | 7,626 | 2,765 | 347,947 |
| S.E. & C. .. .. | 825,294 | 212,085 | 83,545 | 222,345 | 75,035 | 39,262 | 57,660 | 1,515,226 |
| Taff Vale .. .. | 70,992 | 102,713 | 5,032 | 135,133 | 1,802 | 8,366 | 7,694 | 331,732 |
| Total .. .. | 12,010,130 | 9,276,899 | 1,053,798 | 7,683,521 | 762,459 | 1,852,829 | 1,055,652 | 33,695,288 |
| SCOTLAND: | | | | | | | | |
| Caledonian .. .. | 649,065 | 682,234 | 65,326 | 541,790 | 88,790 | 76,004 | 54,330 | 2,157,539 |
| Glasgow & S.W. .. | 275,433 | 230,621 | 27,568 | 155,175 | 28,930 | 42,004 | 16,491 | 776,222 |
| North British .. | 586,514 | 757,912 | 36,445 | 571,137 | 56,875 | 145,309 | 66,075 | 2,220,267 |
| Total .. .. | 1,511,012 | 1,670,767 | 129,339 | 1,268,102 | 174,595 | 263,317 | 136,896 | 5,154,028 |
| Total for Great Britain | **13,521,142** | **10,947,666** | **1,183,137** | **8,951,823** | **937,054** | **2,116,146** | **1,192,548** | **38,849,316** |

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* Compiled from the Ministry of Transport's Monthly "Railway Statistics."

which are serious disadvantages, particularly to the busy commercial man. The convenience of the person who is to use the Table must be consulted, and it must be constructed so as to be easily read, easily understood, and its figures easily compared and followed, without unnecessary waste either of time or thought. Tables which are too large and therefore unwieldy should therefore be split into sections, and these drawn together into a concrete whole by means of a summarised table. No Table should be of such a size that the eye finds it difficult to take the whole in at a glance. When deciding upon the number of columns it is important to place figures which are intended to be compared as near to each other as is practicable, and preferably in vertical formation rather than horizontal, for the eye has greater difficulty in traversing a horizontal line of figures than a vertical column. Where actual figures *and* percentages or averages appear in the same Table, the percentages or averages should be placed in close proximity to the figures upon which they are based. If Totals are the principal figures to be compared, they are best placed at the head of a vertical formation or at the left-hand side of horizontal tabulation, but the reverse procedure is more advantageous when the subsidiary columns are those of primary importance, as is shown in Table K opposite. No hard-and-fast rules on this matter can be laid down. Any departure from the practice adopted is apt to result in confusion to the reader, and hence affect the value of the Table adversely.

In deciding the number of columns into which the matter is to be tabulated due regard must be given to the relative importance of the different data, and the ruling of the Table should closely follow this relative importance. The principal sections should be divided by heavy or multiple rulings, and the breadth of the ruling should vary with the importance of the sub-divisions. It will frequently be found that some of the data do not fall within the divisions decided upon, and in such cases a "Miscellaneous" column becomes a necessity, but care must be taken to see that these examples are not of a widely varying nature, as otherwise footnotes and references will be necessary, and when this happens the Table is not sufficiently self-contained to give the best results. When the Miscellaneous items vary in character and are numerous, it is better to make a subsidiary table for all such items, and tabulate them in such a manner as will obviate the necessity for notes and references to explain them. The divisions into which the matter is divided should be of such a nature that the data are adequately distributed over them, for a Table showing some columns full and others with numerous blank spaces is apt to be confusing to the reader, therefore divisions which contain regularly recurring data should be placed together so that others where the data is irregular in its occurrence do not intervene. Care must also be taken to "range" the columns, *i.e.* place millions under millions, thousands under thousands and so on. This makes the Table more readable and aids comparison and computation. Numerous lengthy columns of numerical data containing a large number of digits to each individual number should, whenever possible, be avoided, and thus approximation should be used provided that the data does not lose any of its value in so doing; for example, it would not have to be carried too far were the Tables intended to be used for reference rather than for drawing reasoned conclusions therefrom.

## § 4.—**Data to be Tabulated.**

Having settled the object for which the Table is to be compiled, the next point to be considered is whether we shall tabulate the actual figures or express them as ratios or averages, and this can only be decided by a careful consideration of the data and the object for which they are being used. The Output of a Factory, the Turnover of a Business, the Imports or the Exports of Goods, the Goods carried by Vans or Wagons, are all examples where the actual figures would be used in tabulation. The Profits of a Company, the Dividends paid to Shareholders, the Number of Hours engaged on Productive Work, would always be better expressed in the form of percentages or ratios, while the Weight of Goods carried per Van or Wagon, the Output of a Factory for each unit of time, are cases where averages can be employed to greater advantage than any other form of figures. If, however, the Tables are to be used for further tabulation or inquiry rather than for comparison, the actual numbers should always be shown, even if they are approximated to a degree which does not affect the inferences to be drawn therefrom, in order that any further inquiry considered necessary or advisable can be carried out easily and correctly. When figures are large and comparison consequently difficult extra columns should be added, so that the percentages, or ratios which the items bear to the total, or the averages prepared on the lines laid down, may be shown. This is illustrated in the following Table :—

### TABLE L.*

Number of Wagon-Miles (*i.e.* Number of Wagons run multiplied by Miles run) of Six of the Principal Railway Companies of Great Britain for the Four Weeks ended 29th February, 1920.

| Company. | Loaded. | Percentage of Total. | Empty. | Percentage of Total. | Total. |
|---|---|---|---|---|---|
| Great Eastern | 14,421,121 | 69·13 | 6,440,450 | 30·87 | 20,861,571 |
| Great Northern | 18,787,862 | 69·58 | 8,212,941 | 30·42 | 27,000,803 |
| Great Western | 42,442,558 | 72·11 | 16,412,734 | 27·89 | 58,855,292 |
| L. & N.W. | 42,520,062 | 74·56 | 14,505,089 | 25·44 | 57,025,151 |
| Midland | 49,768,135 | 68·46 | 22,932,112 | 31·54 | 72,700,247 |
| North British | 16,566,493 | 76·02 | 5,225,596 | 23·98 | 21,792,089 |

* Compiled from the Ministry of Transport's Monthly "Railway Statistics."

## TABLE M.

ANALYSIS OF THE BANK OF ENGLAND WEEKLY RETURN FOR THE TWENTY WEEKS ENDED 8TH SEPTEMBER, 1920.

| Week ended. | | Coin and Bullion. | Circulation excluding Bank Post Bills. | Deposits. | Securities in Banking Department. | Reserve. | % of Reserve to Liabilities. | Bank Rate. |
|---|---|---|---|---|---|---|---|---|
| 1920. | | £ | £ | £ | £ | £ | % | % |
| April | 28 | 112,518,311 | 107,883,945 | 140,381,013 | 134,969,049 | 23,084,366 | 16½ | 7 |
| May | 5 | 112,520,217 | 111,115,815 | 137,165,807 | 135,000,198 | 19,854,402 | 14½ | — |
| | 12 | 112,518,503 | 111,312,935 | 120,833,054 | 118,899,746 | 19,655,568 | 16¼ | — |
| | 19 | 112,472,462 | 110,881,420 | 121,450,053 | 119,147,462 | 20,041,042 | 16½ | — |
| | 26 | 112,455,304 | 111,464,320 | 117,783,506 | 116,072,381 | 19,440,984 | 16½ | — |
| June | 2 | 114,458,784 | 113,806,555 | 148,563,964 | 147,204,839 | 19,102,229 | 12⅞ | — |
| | 9 | 116,706,689 | 114,367,090 | 156,315,017 | 153,289,184 | 20,789,599 | 13¼ | — |
| | 16 | 117,690,113 | 114,165,595 | 153,812,402 | 149,614,935 | 21,974,518 | 14¼ | — |
| | 23 | 117,815,415 | 115,240,110 | 136,343,168 | 133,142,988 | 21,025,305 | 15⅜ | — |
| | 30 | 117,882,041 | 120,057,840 | 191,715,167 | 193,266,862 | 16,274,201 | 8½ | — |
| July | 7 | 120,737,702 | 122,743,815 | 134,921,503 | 136,319,467 | 16,443,887 | 12¼ | — |
| | 14 | 122,879,497 | 124,237,640 | 137,293,937 | 138,061,145 | 17,091,857 | 12½ | — |
| | 21 | 123,004,078 | 124,098,825 | 136,153,359 | 136,669,757 | 17,355,253 | 12¾ | — |
| | 28 | 123,234,383 | 125,369,195 | 133,798,903 | 135,393,570 | 16,315,188 | 12¼ | — |
| Aug. | 4 | 123,108,175 | 126,489,075 | 148,148,252 | 151,054,505 | 15,069,100 | 10¼ | — |
| | 11 | 123,067,552 | 125,527,795 | 140,018,047 | 142,056,014 | 15,989,757 | 11⅜ | — |
| | 18 | 123,081,476 | 124,844,910 | 133,249,143 | 134,625,400 | 16,686,566 | 12½ | — |
| | 25 | 123,028,857 | 124,893,550 | 115,955,156 | 117,438,601 | 16,585,307 | 14¼ | — |
| Sept. | 1 | 123,081,830 | 126,532,520 | 129,772,841 | 132,875,147 | 14,999,310 | 11½ | — |
| | 8 | 123,077,317 | 125,908,565 | 133,489,220 | 135,968,879 | 15,618,752 | 11¾ | — |

## TABLE N.

### Analysis of the Bank of England Weekly Return for the Twenty Weeks ended 8th September, 1920.

| Week Ended. | Coin and Bullion. | Circulation excluding Bank Post Bills. | Deposits. | Securities in Banking Department. | Reserve. | Per Cent. of Reserve to Liabilities. | Bank Rate. |
|---|---|---|---|---|---|---|---|
| 1920 | £ | £ | £ | £ | £ | % | % |
| April 28 .. .. | 112,518,311 | 107,883,945 | 140,381,013 | 134,969,049 | 23,084,366 | 16½ | 7 |
| May 5 .. .. | 112,520,217 | 111,115,815 | 137,165,807 | 135,000,198 | 19,854,402 | 14½ | — |
| 12 .. .. | 112,518,503 | 111,312,935 | 120,833,054 | 118,899,746 | 19,655,568 | 16¼ | — |
| 19 .. .. | 112,472,462 | 110,881,420 | 121,450,053 | 119,147,462 | 20,041,042 | 16½ | — |
| 26 .. .. | 112,455,304 | 111,464,320 | 117,783,506 | 116,072,381 | 19,440,984 | 16½ | — |
| June 2 .. .. | 114,458,784 | 113,806,555 | 148,563,964 | 147,204,839 | 19,102,229 | 12⅞ | — |
| 9 .. .. | 116,706,689 | 114,367,090 | 156,315,017 | 153,289,184 | 20,789,599 | 13¼ | — |
| 16 .. .. | 117,690,113 | 114,165,595 | 153,812,402 | 149,614,935 | 21,974,518 | 14¼ | — |
| 23 .. .. | 117,815,415 | 115,240,110 | 136,343,168 | 133,142,988 | 21,025,305 | 15⅜ | — |
| 30 .. .. | 117,882,041 | 120,057,840 | 191,715,167 | 193,266,862 | 16,274,201 | 8½ | — |
| July 7 .. .. | 120,737,702 | 122,743,815 | 134,921,503 | 136,319,467 | 16,443,887 | 12¼ | — |
| 14 .. .. | 122,879,497 | 124,237,640 | 137,293,937 | 138,061,145 | 17,091,857 | 12½ | — |
| 21 .. .. | 123,004,078 | 124,098,825 | 136,153,359 | 136,669,757 | 17,355,253 | 12¾ | — |
| 28 .. .. | 123,234,383 | 125,369,195 | 133,798,903 | 135,393,570 | 16,315,188 | 12¼ | — |
| Aug. 4 .. .. | 123,108,175 | 126,489,075 | 148,148,252 | 151,054,505 | 15,069,100 | 10¼ | — |
| 11 .. .. | 123,067,552 | 125,527,795 | 140,018,047 | 142,056,014 | 15,989,757 | 11⅜ | — |
| 18 .. .. | 123,081,476 | 124,844,910 | 133,249,143 | 134,625,400 | 16,686,566 | 12½ | — |
| 25 .. .. | 123,028,857 | 124,893,550 | 115,955,156 | 117,438,601 | 16,585,307 | 14¼ | — |
| Sept. 1 .. .. | 123,081,830 | 126,532,520 | 129,772,841 | 132,875,147 | 14,999,310 | 11½ | — |
| 8 .. .. | 123,077,317 | 125,908,565 | 133,489,220 | 135,968,879 | 15,618,752 | 11¾ | — |

When it becomes necessary to use actual figures, and these figures not only contain numerous digits, but are also large in number and tabulated in various columns, the difficulty of tracing the items on the same lines in such a compact mass of figures becomes very great, and the consequent confusion detracts considerably from the value of the Table, and, moreover, the resultant tabulation does not appear attractive even to those whose business it is to study such returns. The Table shown on p. 84 is a summary of the Bank of England Return for twenty weeks in the form presented by the *Economist.*

This Table is fairly easy to read as the number of columns and the items are not very numerous, but as the number of columns and/or the number of items increases, the difficulty of reading the Table becomes greater and its value therefore less. Much, however, can be done to improve such masses of figures by care in the printing of them—by slightly widening the columns, by heavier division lines, and by leaving a blank space after each few lines (say after those figures relating to each month, or after every fifth line). The Table would then appear as on p. 85, and it will at once be seen that it is not only more readable, but its appearance is much more attractive.

The compiler must, however, study his problem carefully and plan his table to meet the particular needs of each situation. A draft of the form should always be prepared before proceeding with the work of inserting the figures.

## § 5.—Importance of Checking Calculations.

Whenever calculations are necessary, great care should be exercised to check the correctness of each stage of the work, preferably by some person other than the one who made the original calculation, for, if errors be discovered by the reader of the Table, doubt is thrown upon the whole of the Table, and its utility is destroyed or greatly impaired.

It is particularly necessary to see that percentages are correctly worked, and averages prepared upon the correct bases, and alternative checks should be used whenever possible. When working percentages of various items of the same base the calculations can be checked by adding the percentages, since they should collectively amount to 100 per cent.; for instance:—

| Branch. | Amount of Turnover. | Percentage of Total. |
|---|---|---|
| | £ | Per Cent. |
| A .. .. | 60,000 | 20 |
| B .. .. | 75,000 | 25 |
| C .. .. | 120,000 | 40 |
| D .. .. | 45,000 | 15 |
| **Total ..** | **300,000** | **100** |

It must, however, be remembered that when the percentages are worked out to fractions, and decimals are used correct to a defined place, it may happen that the result will not exactly equal 100 per cent., though it will always be within a very small fraction thereof.

If, however, the basis upon which the percentages are calculated vary with the different items, no such result can be obtained, and care must, moreover, be taken to see that an error is not made by "taking the average of an average." This is illustrated in the following example:—

| Branch. | Turnover. | Expenses. | Percentage of Expenses to Turnover. |
|---|---|---|---|
| | £ | £ | Per Cent. |
| A .. .. | 60,000 | 36,000 | 60 |
| B .. .. | 75,000 | 60,000 | 80 |
| C .. .. | 120,000 | 90,000 | 75 |
| D .. .. | 45,000 | 30,000 | 66⅔ |
| **Total ..** | **300,000** | **216,000** | **72** |

To obtain the percentage which the total expenses of the business bear to the total Turnover it is necessary to work with the totals, *i.e.* $\frac{216,000}{300,000} \times 100 = 72$ per cent., whereas a careless non-mathematical worker might add the percentages together and divide by the number of items, *i.e.* $\frac{60 + 80 + 75 + 66\frac{2}{3}}{4} = 70{\cdot}416$ per cent., a result which is entirely wrong, and which if used for subsequent work would lead to further serious errors.

## § 6.—Title of the Table.

Having decided the use to which the Table is to be put, and the figures to be used to obtain the desired result, the next point of importance will be the **Title** of the Table. Since the principal idea in preparing Tabular Statements is to place data before people who have taken no part in their preparation, it is above all things necessary that the Table be entirely *self explanatory*, and for this reason great care must be exercised in the choice of the Title. This must be brief, but it must not be so brief that its meaning becomes obscure or ambiguous. Due attention must also be paid to the Headings of the sub-divisions and columns, and when these are clear and well defined no direct reference need be made to them in the main title. The Title should therefore be wide in scope as well as clear of meaning. Wherever possible, the sub-divisions should be arranged in vertical form, and follow some well defined and easily understood order, *e.g.* alphabetical, geographical, size or merit, the horizontal columns being utilised for units of measurement when these are present. This form cannot, of course, be always followed, but experience will always show the Table most suited to the needs of the particular business.

In the general arrangement of the Table much can be done to assist the reader by careful selection of type matter.

## TABLE O.*

**Tonnage of Freight Traffic (including Live Stock and Free-hauled Traffic other than in Departmental Trains) originating on or passing over the Undermentioned Railways during the Four Weeks ended 29th February, 1920.**

| Railway Company. | Total. | | General Merchandise (including Livestock). | | Coal, Coke and Patent Fuel. | | Other Minerals. | |
|---|---|---|---|---|---|---|---|---|
| | Including Free-hauled. | Excluding Free-hauled. | Including Free-hauled. | Excluding Free-hauled. | Including Free-hauled. | Excluding Free-hauled. | Including Free-hauled. | Excluding Free-hauled. |
| England and Wales : | Tons. | Tons. | Tons. | Tons. | Tons. | Tons. | Tons. | Tons. |
| Great Central .. | 2,576,047 | 2,484,689 | 483,330 | 474,562 | 1,578,324 | 1,505,298 | 514,393 | 504,829 |
| Great Eastern .. | 1,163,521 | 1,058,173 | 535,231 | 525,552 | 488,091 | 400,707 | 140,199 | 131,914 |
| Great Northern .. | 1,987,623 | 1,909,791 | 563,164 | 550,643 | 1,123,854 | 1,066,683 | 300,605 | 292,465 |
| Great Western .. | 4,876,167 | 4,679,758 | 1,072,633 | 1,067,344 | 2,970,644 | 2,813,191 | 832,890 | 799,223 |
| L. & Y. .. .. | 2,202,285 | 2,077,406 | 694,121 | 678,156 | 1,265,657 | 1,161,410 | 242,507 | 237,840 |
| L. & N.W. .. .. | 4,677,523 | 4,461,006 | 1,335,729 | 1,320,760 | 2,433,397 | 2,243,856 | 908,397 | 896,390 |
| L. & S.W. .. .. | 667,723 | 621,273 | 285,652 | 280,484 | 274,804 | 237,368 | 107,267 | 103,421 |
| L.B. & S.C. .. | 387,298 | 371,451 | 133,109 | 127,298 | 176,220 | 169,394 | 77,969 | 74,759 |
| Midland .. .. | 4,596,803 | 4,404,336 | 1,065,801 | 1,032,160 | 2,604,612 | 2,469,971 | 926,390 | 902,205 |
| North Eastern .. | 5,031,835 | 4,882,274 | 939,120 | 921,548 | 2,907,231 | 2,804,449 | 1,185,484 | 1,156,277 |
| North Staffs .. | 576,144 | 563,782 | 150,395 | 148,661 | 267,667 | 259,372 | 158,082 | 155,749 |
| S.E. & C. .. .. | 747,197 | 730,907 | 256,129 | 249,453 | 392,247 | 386,558 | 98,821 | 94,896 |
| Taff Vale .. .. | 1,169,464 | 1,156,943 | 81,794 | 79,639 | 1,057,318 | 1,047,897 | 30,352 | 29,407 |
| **Total .. ..** | **30,659,630** | **29,401,789** | **7,596,208** | **7,456,260** | **17,540,066** | **16,566,154** | **5,523,356** | **5,379,375** |

* Compiled from the Ministry of Transport's Monthly " Railway Statistics."

The main heading should preferably be in the Roman type, and the size of the type used should vary with the importance of the divisions and sub-divisions, all headings of equal importance being printed in the same style and size of type. Generally speaking, a Typewriter is not suitable for tabulated statements, owing to the fact that the style and size of the characters is fixed to one type; but where the Tables follow a pre-arranged form, and the form is a printed one, the Typewriter may be used with advantage in filling in the printed form—particularly as by means of a Tabulating attachment the figures can be correctly ranged, *i.e.* units under units, tens under tens and so on. An adding attachment will also save much time in checking Totals.

The Tabular Statement on page 89 pays due attention to all these points, and may therefore be taken as a model.

## § 7.—**Tabulation of Commercial Data.**

The use of Statistical matter need not necessarily be confined to the inside work of the office, but may frequently be used to support arguments advanced to customers or prospective customers. The advertisement on the opposite page, taken from a recent advertisement in an American paper, illustrates the uses to which statistical records may be put.

From the point of view of the Statistician the statement, though clear and lucid, has several weak points. An instance of one is the inclusion of the number of days operated by the two trucks without assigning reasons for the wide divergence which appears in the number of days worked, as it is hardly possible that the time the Truck numbered 44 was not working was directly attributable to tyre trouble, though if such happened to be the case the statement would be in order. The comparison of the miles travelled is also fallacious, in so far that the number of days worked by the Trucks was unequal, that is unless tyre trouble or other troubles directly due to the tyres are responsible for Truck numbered 44 being off the road. If we take the average mileage covered per day, we find that Truck No. 44 did $\frac{5,388}{130}$, or 41·45 miles per day, whereas No. 45 averaged $\frac{6,719}{146}$, or 46 miles per day. This gives an increase in favour of the latter of 10·97 per cent. a very different result from the 25 per cent. increase shown in the Table. If, however, the Statement showed the reason for the difference in the number of days worked, criticism would not be so likely to arise.

ADVERTISEMENT.

**Beginning May 1, 1919, a six months' test of solid versus pneumatic truck tyres was conducted by Mandel Brothers, operating a large downtown dry goods store in Chicago. The test was supervised by the company's chief engineer, Mr. Alfred Johnson. Two trucks were used, No. 44 on solid tyres and No. 45 with Goodyear Cord Tyres on the rear wheels and other pneumatics on the front wheels. Both trucks were employed in regular city delivery work, handling similar loads over similar routes. The results of the test, given below, have caused the company to order recently several new trucks on pneumatics.**

| | Truck No. 44 (Completely equipped with Solid Tyres). | Truck No. 45 (Completely equipped with Pneumatic Tyres). | Difference in Favour of Pneumatic Equipment. |
|---|---|---|---|
| Number of days operated | 130 | 146 | 12·3%increase |
| Miles travelled | 5,388 | 6,719 | 25 % increase |
| Miles per gallon of gasoline | 5·5 | 7·2 | 31 % increase |
| Repairs per mile | $ ·0109 | $ ·0057 | 47 % saving |
| Labour Cost per mile (Drivers) | $ ·20 | $ ·176 | 12 % saving |
| Operating Cost per mile | $ ·353 | $ ·335 | 5 % saving |

NOTE.—The actual saving in operating cost amounted to 1·8 cents per mile. Such a reduction grows to a very substantial sum of money when multiplied by thousands of miles of service.

This illustration will serve to emphasise the necessity for careful preparation of Statistical matter, and the elimination of all matters of doubt, and the substitution of data or headings which will prevent criticism and avoid confusion or doubt.

## SYNOPSIS TO CHAPTER VI.

# THE GRAPHIC METHOD.

§ 1.—APPLICATION OF THE GRAPHIC METHOD.

2.—THE FREQUENCY POLYGON.

3.—THE NORMAL FREQUENCY CURVE.

4.—THE OGIVE.

5.—KINDS OF DIAGRAMS.
  (*a*) The "Block" Diagram.
  (*b*) The "Bar" Diagram.
  (*c*) The Graph.
    (1) The Natural Scale Graph.
    (2) The Logarithmic Scale Graph.
  (*d*) The "Bar" Diagram to show Components
  (*e*) Comparisons of Areas and Volumes.
  (*f*) Diagrams to show Three Factors.

6.—RULES FOR CONSTRUCTING DIAGRAMS.

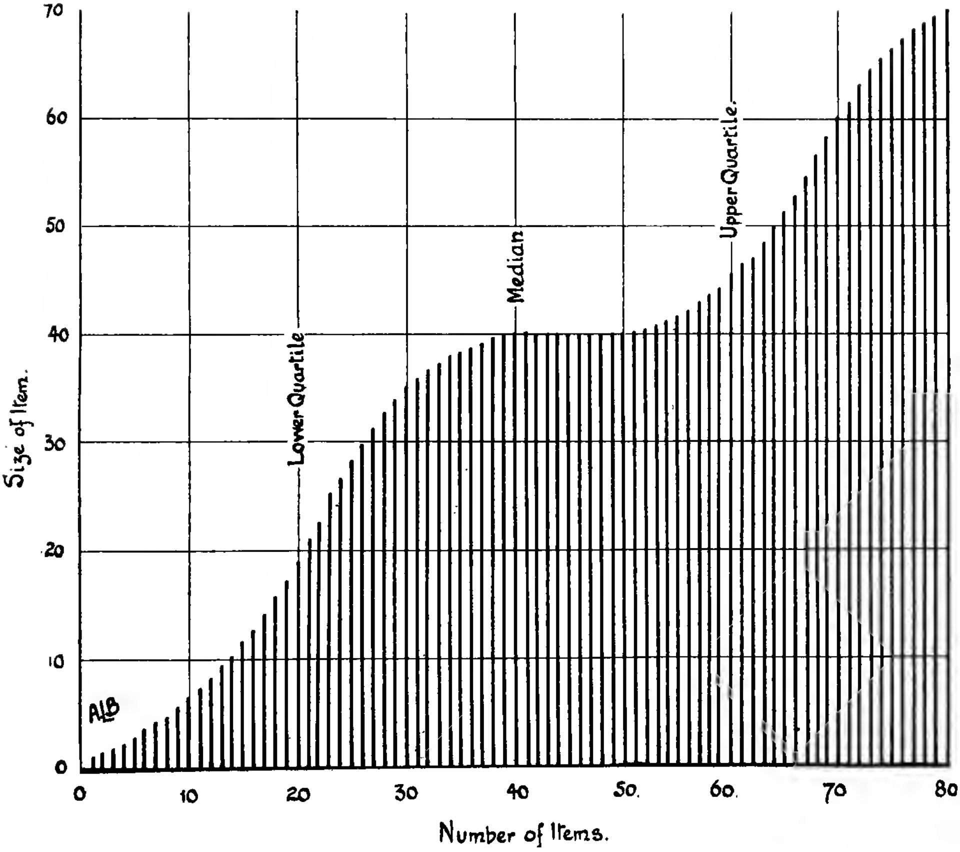
## CHAPTER VI.

# THE GRAPHIC METHOD.

To the trained mind a carefully compiled Table is of great value, both for purposes of reference and for deductive reasoning, but if Statistical methods are to become of really great utility to the business man who is not a skilled mathematician, it behoves us to present our data and facts in such a manner that not only are the details supplied, but the entire meaning and trend of the data presented can be seen without the necessity for long consideration and study. It must be frankly admitted that Tabulation falls very short of our Standard in this respect, for though the meaning can be grasped by one with mathematical training, it only presents a mass of figures to others. Tabulated statements, however carefully prepared, are not attractive to the business man, who finds the study of them tedious and involved, and frequently finds it necessary to call for explanations and reasonings from those responsible for their compilation. It is difficult for those not accustomed to mathematics to visualise conditions and results from mathematical formulæ and data. The actual trend of the figures is frequently obscured when the data is presented in Tabular form, and the eye is frequently tired by referring to masses of figures. Pictorial Representation, when properly carried out, appeals to the eye, and also to the mind, because it is practical, clear, and easily understandable even by those unacquainted with the method of preparation. A statement that a series of articles are arrayed in order of magnitude with the smallest on the left and the largest on the right does not convey the same result to the mind as is obtained by glancing at the same facts presented in Diagram No. 1. While an effort has to be made to visualise the position when only the written statement is available, a mere glance at the diagram is sufficient to make the meaning clear, and time is thus saved to the busy man as a consequence, and the chance of errors is also reduced considerably. A busy physician visiting patients in hospitals obtains a very easily-read clue to the progress and condition of his patient by a glance at the chart of the temperature which usually hangs at the head of each bed. This chart tells him the changes which have taken place since his last visit, and eliminates the necessity for inquiry as to what has happened in his absence. The wandering of a line is more powerful in its effect on the mind than a tabulated statement; it shows what is happening, and what is likely to take place, just as quickly as the eye is capable of working. The graphic method so far has not been greatly used in commerce, but its utility is being increasingly recognised, for its advantages are many; but few people have been accustomed to read graphs, and thus the majority do not appreciate their usefulness. As with Tabulation, great care must be taken in the preparation of diagrams and graphs, though they are capable of being adapted to any circumstances and needs which may arise. *Diagram No.* 1 shows the simplest possible form of diagram. It is easy of compilation, requires no particular skill or training to construct, is easily read and understood, and the significance of the various factors is at once apparent. In spite of these advantages it is not an ideal method of diagrammatic presentation, for it involves considerable work, and great care is needed in drawing the many lines in the data; while if these lines become very numerous it would be necessary either to enlarge the diagram,

## DIAGRAM No. 1.

SHOWING A SERIES OF EIGHTY ITEMS ARRAYED.



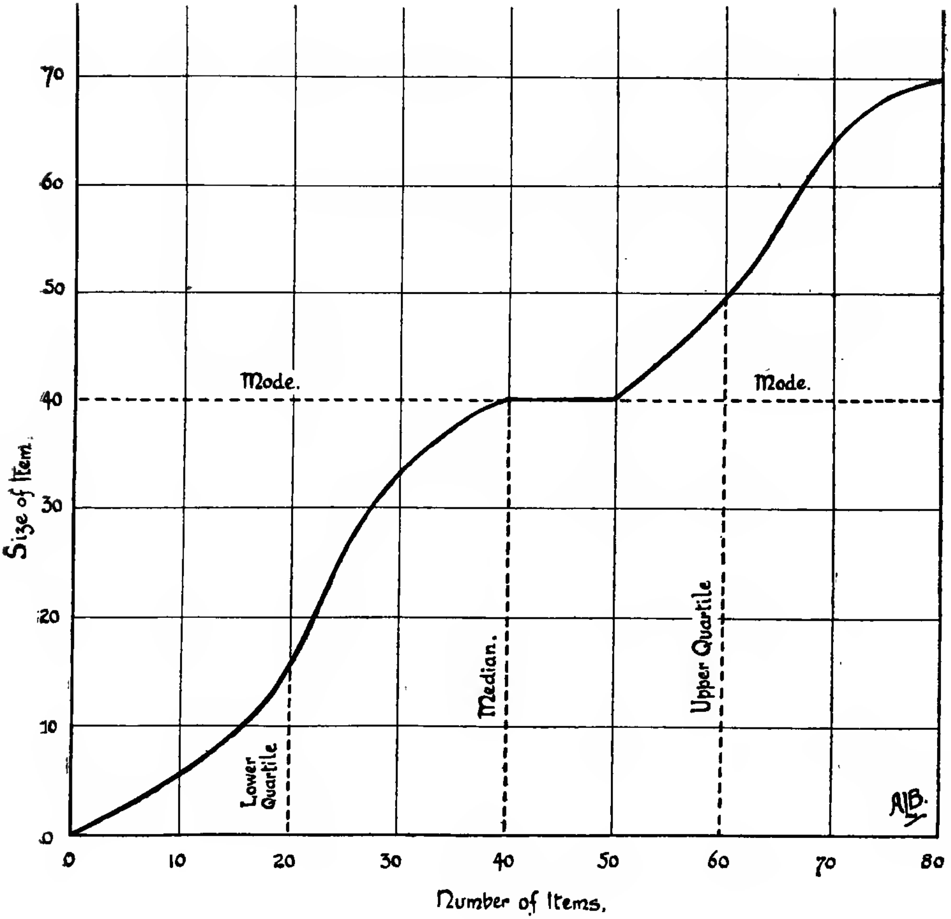
or else to draw the lines closer together, and either of these alternatives would result in the eye becoming confused with the multitude of lines, the diagram consequently declining in value. Such data are always better plotted in the form of a curve, as is shown in *Diagram No.* 2. The data are marked on the sectional paper in the same manner as in the first example, but, instead of drawing lines from the base line to the points so obtained, these latter are joined together by a curve which gives exactly the same outline as Diagram No. 1. Owing to the absence of the numerous lines the varying sizes of the items can be traced more clearly than in the first diagram, and the representation is much clearer. Such a curve is easily plotted and drawn, and, moreover, if the items are more numerous than those shown, the horizontal scale can be adjusted and the curve re-drawn without the chart becoming either more unreadable or confusing, and this is a very distinct advantage. Though this particular type of diagram (*i.e.* the plotting of an array of the data) is not likely to be met with to any degree in the commercial world, the principle upon which it is prepared is an extremely useful one, and capable of almost infinite variations, being thus adaptable to any particular data or problem which may confront one.

## § 1.—**Applications of the Graphic Method.**

Before considering the utility of the graph to the business man it will be well to consider how it is applied to the science of Statistics, and the lessons thus learnt can be applied to the everyday problems of the Commercial world. We have already seen that if we collect data of any description and grade them there will be found a tendency for the items to fluctuate around a particular type which we already know as the Mode. This is true of all chance and natural phenomena. The majority of the items will be found to be

DIAGRAM No 2

Curve showing "Array" of Eighty Items of varying Size.



grouped near this mode, and as the distance from the mode increases in either direction the number of items becomes less, and, in the case of natural phenomena, a limit of fluctuation can usually be approximately fixed. As has already been seen in Diagram No. 1, and as is shown even more clearly in Diagram No. 2, the Mode is located in this series of data between the fortieth and forty-ninth items, since they are all of the same magnitude and the curve "flattens" out at this point in the array after rising steeply and continuing with equal steepness. Experience shows that it will make no difference to our result if we increase the number of items under review, *provided that those samples we have taken and plotted are truly representative of the whole of the examples*, and it will thus be seen that the correctness of our work in the early stages of the compilation is of supreme importance if our deductions are to be of real use to us.

Usually, however, it will be found that our data do not take the simple form shown in the diagrams numbered 1 and 2, but many of the items are of equal dimensions, and when this is the case it becomes necessary to compile and plot a frequency table, and it is then essential that the class interval (*i.e.* the variations in the magnitude of the data) should be uniform. Let us assume that a census has been made of the businesses of a particular kind of trade, and from the results we have selected 335 examples where the capital involved is the same, but the profits earned vary from £1 to £4,500. Here the variation between the smallest and the largest examples is very wide, and we get a large number of items between the two extremes varying so little in magnitude that to plot them all correctly would mean a very large and cumbersome diagram. In such cases the only thing to do is to take arbitrary dividing lines and collect the examples into groups, taking care that the class interval is the same in each case. Let us assume that the following simple Frequency Table is obtained as a result of such grouping :—

TABLE P.

TABLE SHOWING FREQUENCY WITH WHICH CERTAIN DATA OCCURS.

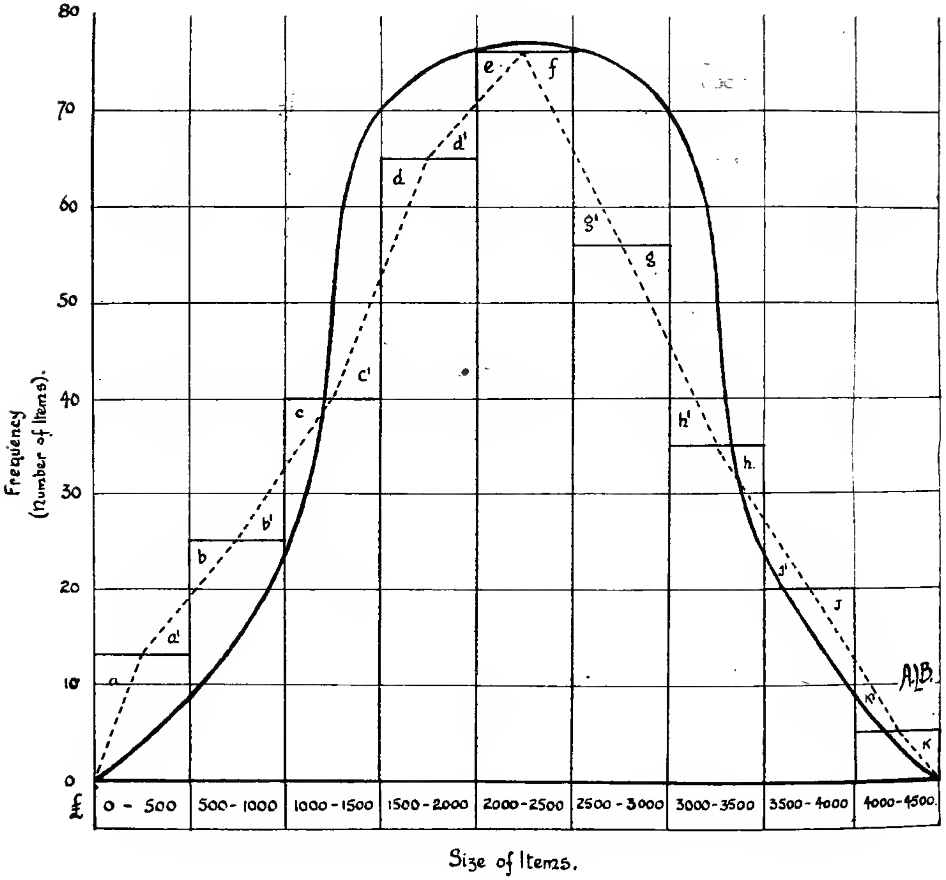
| **Size of Item.** (Amount of Profits.) | | | | **Frequency.** (Number of Examples in Each Group.) |
|---|---|---|---|---|
| Not exceeding £500 .. | | | | 13 |
| Exceeding | £500 | but not | £1,000 .. | 25 |
| ,, | £1,000 | ,, | £1,500 .. | 40 |
| ,, | £1,500 | ,, | £2,000 .. | 65 |
| ,, | £2,000 | ,, | £2,500 .. | 76 |
| ,, | £2,500 | ,, | £3,000 .. | 56 |
| ,, | £3,000 | ,, | £3,500 .. | 35 |
| ,, | £3,500 | ,, | £4,000 .. | 20 |
| ,, | £4,000 | ,, | £4,500 .. | 5 |

## § 2.—The Frequency Polygon.

This information would then be plotted as in Diagram No. 3, with rectangular blocks to represent the numbers in each group. As the class boundaries are chosen in an arbitrary fashion it follows that the rectangular diagrams would be entirely different were the examples grouped with other boundaries instead of those shown. If the groups were, for instance, to be limited to intervals of £250, the rectangles would be narrower and the height of the steps would decrease, and if this process were continued sufficiently far a smooth curve would result, but such a result would cause nearly as much work as plotting the actual magnitudes of all the examples. We can, however, approximate the result which would be obtained by plotting each individual item by connecting the outer points of the base of the diagram (*i.e.* 0 and £4,500) with the middle points at the *top* of each of the rectangles as shown by the dotted line on Diagram No. 3. The resultant figure is known as a "**Frequency Polygon**" and it will be seen that the area

DIAGRAM No 3.

Diagram showing Frequency when Items are grouped.



of such Polygon is approximately the same as that of the rectangles. In the diagram under consideration the triangles outside the new figure which are included in the original area roughly correspond in area with those now included, but which were previously excluded (compare *b* and $b^1$, *c* and $c^1$, etc.), but it will be noticed that the triangles *e* and *f* are excluded entirely, and that the area of the Frequency Polygon is therefore a trifle smaller than that of the area enclosed by the rectangles. It must also be recognised that other items might possibly occur outside the limits chosen, *e.g.* a case might occur of profits amounting to more than £4,500, while if some other class interval had been taken there is a possibility that the rectangle for the entire group might be higher, and as a consequence the apex of the polygon might be above that now shown.

## § 3.—The Normal Frequency Curve.

Bearing these facts in mind, we can now draw a final smooth curve, and this will show what is believed to be the actual distribution of the profits earned in that trade with that particular amount of capital, assuming that all the examples in existence were actually included. This curve will be seen from the diagram to be bell-shaped, and is known as the **Normal Frequency Curve,** or occasionally as the **Normal Curve of Error.** In constructing such a curve we endeavour to eliminate all accidental variations which appear in the samples taken and plotted, and thus obtain a result which in all normal circumstances might be regarded as the correct distribution of the data. Such a bell-shaped curve can be constructed for *all* chance or natural phenomena, but in other data, such as those appertaining to economic or sociological investigations, the examples may display such wide irregularities that we may find it impossible to obtain such a Normal Curve of Error. This would to a very large extent be true also of commercial data, for though two businesses in the same trade may be commenced at the same time, with the same capital, and with the same prospects of success, the results may be, and often are, widely divergent, such divergence being due to one cause or a combination of many causes of a varying nature, such as a better business position, greater personal skill of the proprietor, better methods of management, more facilities for credit, etc.

When plotting our data it is not always necessary to plot the rectangles first, as a point can be plotted which would be identical with the middle position at the top of the rectangle, and thus the first thing to appear would be the Frequency Polygon. In drawing the smoothed curve care must be taken to see that the curve changes direction as little as possible, and all irregularities should be smoothed out. The extent to which the smoothing will be carried will depend upon the data involved, and no hard and fast rules can be laid down upon the matter; but since almost every smoothed diagram representing a continuous series will commence with an extremely small number of instances, and after reaching its maximum decrease slowly to zero, the curve should naturally begin and end on the base line.

It may sometimes happen that the information at our disposal is of such a nature that the actual magnitudes of the examples can be plotted, and we shall find that if we pursue similar methods we shall obtain a very similar curve. Let us assume that the frequency table on the opposite page shows the *actual* profits earned in the 335 cases previously taken. From these data we can construct a Frequency Line or Bar Diagram as shown in Diagram No. 4. The size of the items (in this case the profits earned) are plotted on the horizontal scale, while the frequency with which they occur is placed on the vertical scale, and a line drawn from the base line to the point on the vertical scale corresponding to the number of cases of that particular magnitude which occur in the samples under review. These lines are those marked *a* in the diagram. A study of the diagram makes it apparent that the modal profit is £2,250, for there are no less than forty cases in the Table under review. If the law which has been enunciated above applies, as it will to some extent at least, it would follow that if these are really representative examples of the whole, then we may expect that all other examples would follow the same lines, but to guard against error in our assumptions we usually smooth the diagram by drawing a curve to allow for any possible contingencies. When this has been done it will be seen that the curve is again of a bell shape. In the case illustrated in Diagram No. 3 we were aware that there were intermediate examples, but in this case there are no examples known which fall between those plotted, consequently in this case the curve must not be interpreted as showing the

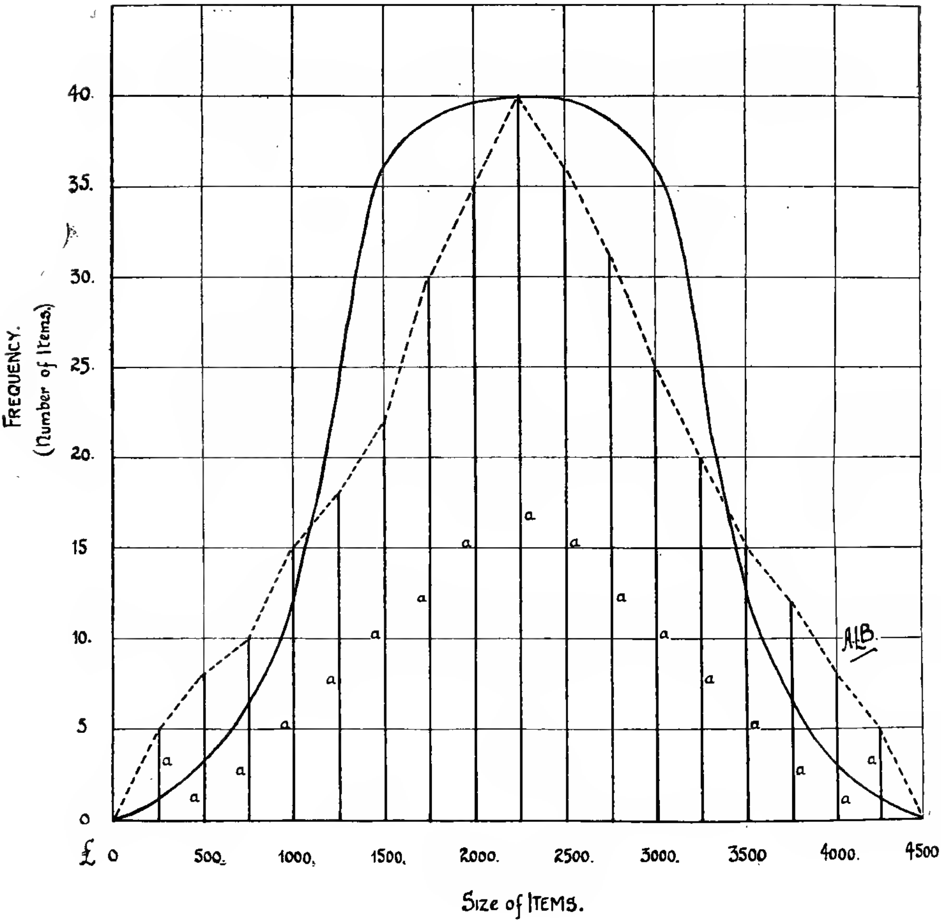
TABLE Q.

TABLE SHOWING FREQUENCY AND CUMULATIVE FREQUENCY OF DATA.

| **Size of Item.** (Amount of Profits.) | **Frequency.** (Number of Examples.) | **Cumulative Frequency.** |
|---|---|---|
| £ | | |
| 250 | 5 | 5 |
| 500 | 8 | 13 |
| 750 | 10 | 23 |
| 1,000 | 15 | 38 |
| 1,250 | 18 | 56 |
| 1,500 | 22 | 78 |
| 1,750 | 30 | 108 |
| 2,000 | 35 | 143 |
| 2,250 | 40 | 183 |
| 2,500 | 36 | 219 |
| 2,750 | 31 | 250 |
| 3,000 | 25 | 275 |
| 3,250 | 20 | 295 |
| 3,500 | 15 | 310 |
| 3,750 | 12 | 322 |
| 4,000 | 8 | 330 |
| 4,250 | 5 | 335 |
| 4,500 | 0 | 335 |

## DIAGRAM No. 4.

Diagram showing Frequency with which Items occur.



existence of intermediate examples as in the last case, *but only as a curve of normal tendency*, and in reality the curve therefore represents the relative height to which the vertical lines would extend were the number of examples infinite.

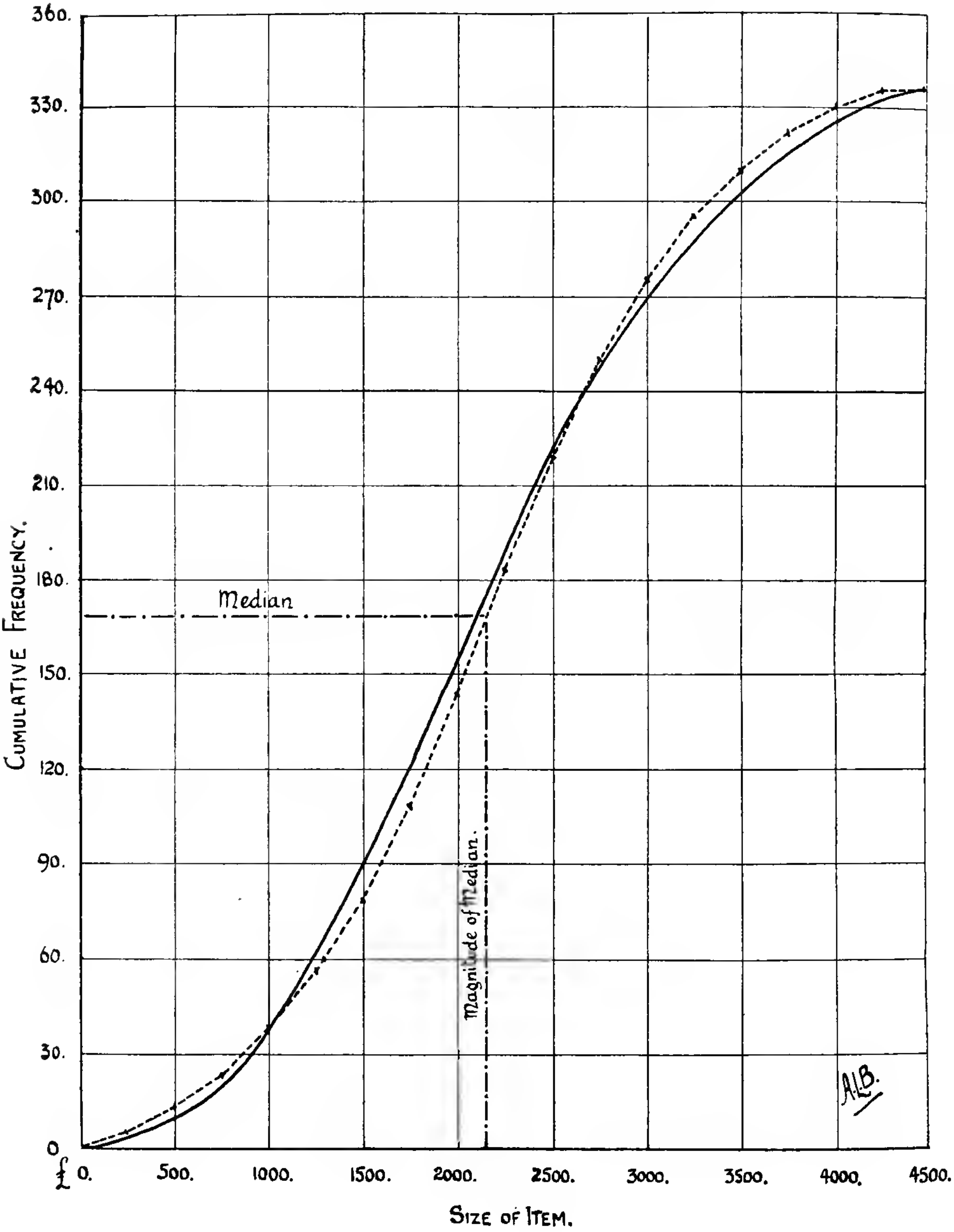
The advantages of such a curve of Normal Frequency are numerous, for by its aid we can visualise what is happening to particular sections of trade and commerce, and so estimate the possibilities of profit from such sections. If such curves were obtainable and were carefully studied by those commencing business, the result would be that speculation would be to a large extent eliminated, and efforts made to obtain the modal result shown by the graph, instead of remaining satisfied with a smaller return. This would lead to more efficient management.

## § 4.—The Ogive.

Another type of diagram used for the consideration of data into which no time element enters is illustrated in Diagram No. 5, and the resultant curve is known as an "**Ogive.**" This is obtained by plotting the cumulative frequency of a series of observations, and in this particular example the cumulative frequency of the profits enumerated in Table Q above have been taken. The dotted line shows the original Ogive plotted from the actual figures, while the continuous thick line is the smoothed curve resulting from the plotting of the data in question. It is usually much easier to smooth an Ogive than a Frequency Polygon, as in Diagrams 3 and 4, and the Ogive is very convenient for locating the Median Quartiles, Deciles, etc. To find the magnitude of the Median it is necessary to locate the number on the Cumulative Frequency Scale (*i.e.* the vertical scale) and draw a horizontal line from this point to the curve, and then from the point so obtained drop a vertical line to the horizontal base line (or axis), and the point where the vertical line intersects such base will give the magnitude

DIAGRAM No 5.

"OGIVE" to show Cumulative Frequency.



of the median example. By means of such graphs the size of the median can be accurately obtained even when the data is indefinite in some respects, and this method is easier than the arithmetical calculation previously explained in Chapter IV. It will be observed that we cannot ascertain the arithmetic Average of the series by inspection of either of the curves appearing in Diagrams 2, 3, 4 or 5, whereas the Median, etc., are easily located and their magnitude measured by a simple mechanical process which does not lend itself to error. The Mode cannot be located easily on the Ogive, but would be at that part of the curve which is steepest.

## § 5.—Kinds of Diagrams.

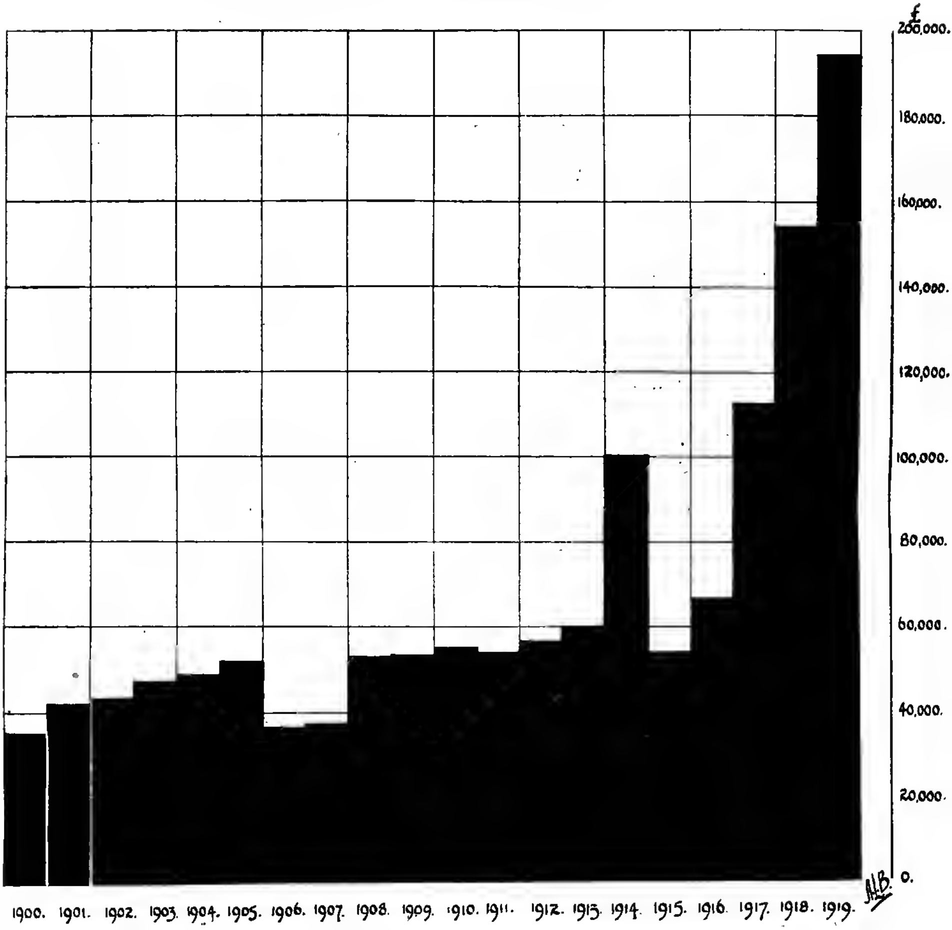
So far we have only considered that form of data into which no element of time enters, but in considering most of the forms of commercial data, particularly when comparison becomes necessary, time enters into the data and plays a conspicuous part, and hence our graphs must take a somewhat different form.

### (a) The "Block" Diagram.

We have recently seen efforts made at presenting commercial data in graphic form, particularly in connection with company flotations, but so far no effort at uniformity in method has been made, and many of the diagrams are faultily conceived and badly executed. Diagram No. 6 is based upon the method used in one such published diagram, and may be termed the "Block" method of presentation. This diagram shows the Turnover of a business for twenty years as displayed on page 52, and though it is a step in the right direction, the diagram is clumsy both in conception and execution. It is difficult to read, for since the co-ordinate lines are obscured, the eye has great difficulty in following the heights of the various rectangles to the scale which appears on the right-hand side of the diagram.

## DIAGRAM No 6.

Diagram showing Annual Turnover for Twenty Years.



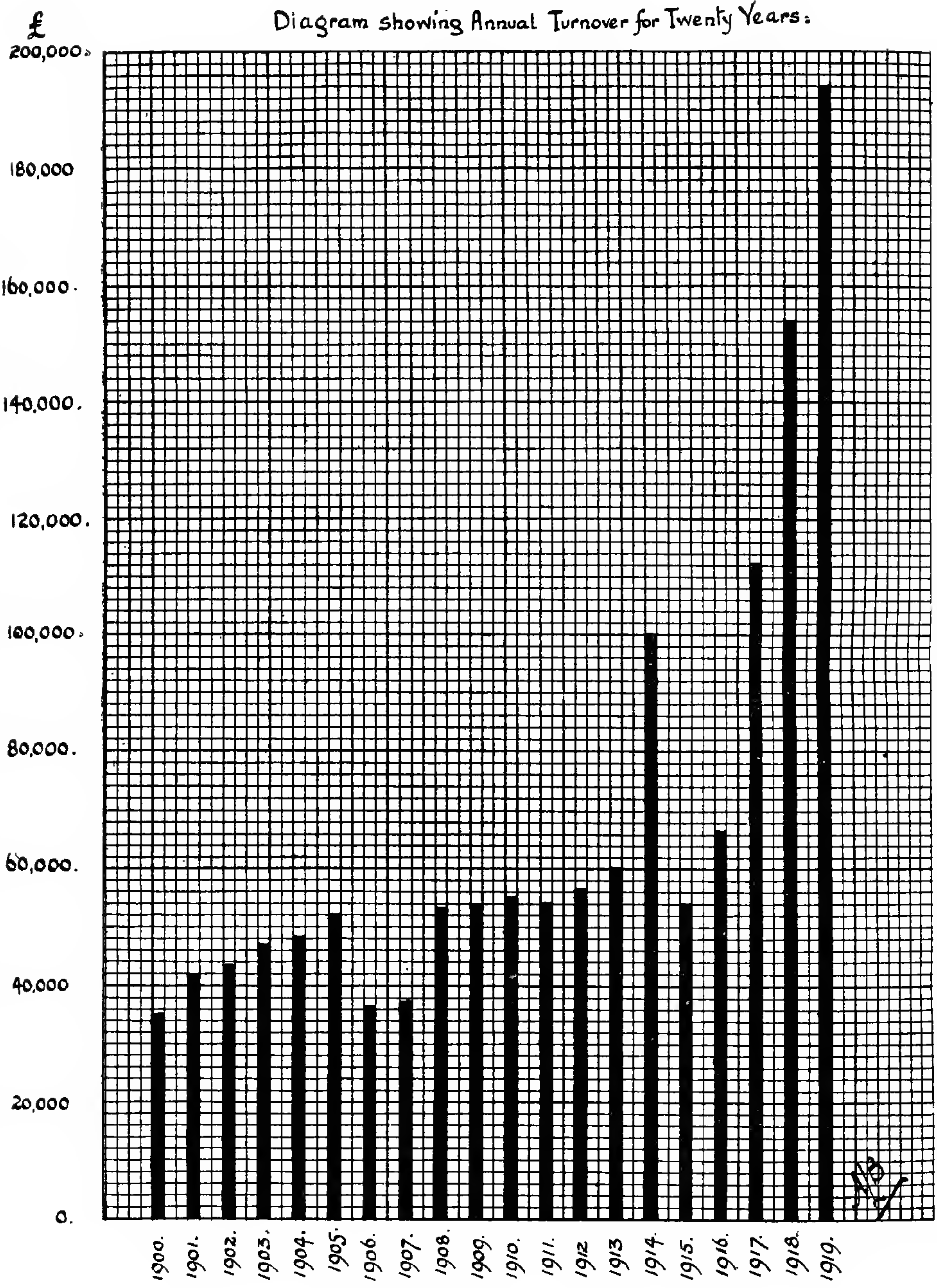
This scale, too, is wrongly placed, since it is always more convenient to print it upon the left-hand side of the diagrammatic matter, so that anyone approaching it knows beforehand what the scale is. The left-hand side scale is more logical in every way, since we always read from left to right, though when the diagram is a wide one it is an aid to reference to give also a scale on the right-hand side. The dividing lines between the various periods of time are indistinguishable in the lower part of the diagram, and this is also a disadvantage. This particular point may be overcome by showing the dividing lines in white, as has been done on the left-hand side of the diagram ; but this entails more work, and from a commercial point of view the diagram must be capable of speedy as well as easy preparation. Such a diagram as this does not lend itself to easy comparison, nor can any other factors be shown on the same chart in order that they may be compared with the main factor.

### (b) The "Bar" Diagram.

In Diagram No. 7 we see the same data plotted in such a manner as to show each year's Turnover as a separate bar, and a comparison of this diagram with No. 6 will at once show that it is not only clearer and more distinct, but that the magnitude of the Turnover is more easily ascertainable from the scale, while the relationship between the Turnover of the various years is more apparent, and the whole diagram is more compact, and hence more useful. This type of diagram, which follows similar lines to the method used in No. 1, is one which is easily understandable by anyone to whom a Table of figures would not appeal. The fluctuations, however small, are clearly shown, and the general result is more attractive than that presented in the previous diagram, but it has the disadvantage also that only one factor can appear at one time ; thus when

## DIAGRAM No. 7.

Diagram showing Annual Turnover for Twenty Years.



comparison becomes necessary additional charts have to be prepared, and then laid side by side. It would, for instance, be impossible to insert the Moving Average, and so eliminate the short-term fluctuations in order to obtain the Trend.

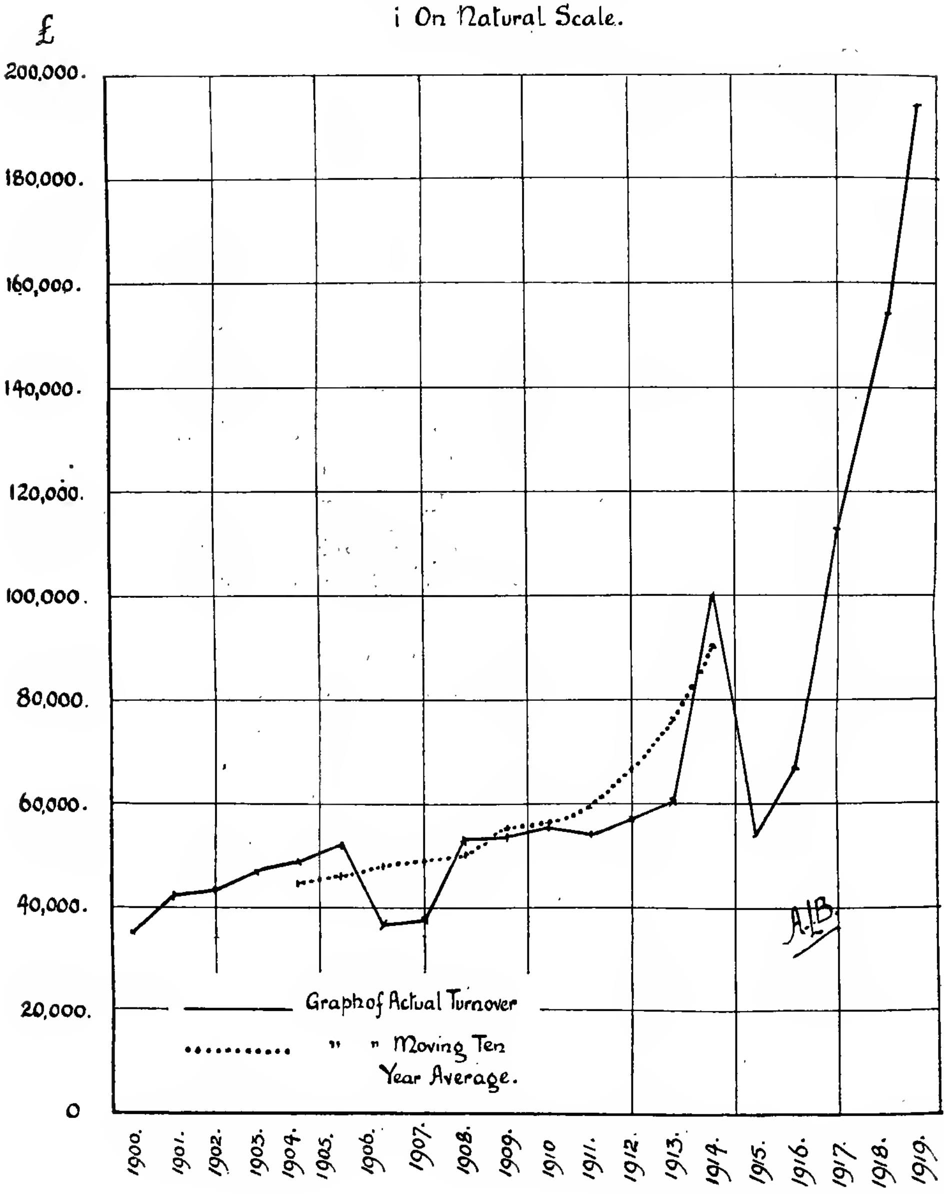
### (c) The Graph.

It follows, therefore, that for all practical purposes we are compelled to use the Curve, since this enables us to obtain the best results, and, moreover, allows space for other curves to appear on the same diagram for comparison. Whichever of the three methods shown is used, the initial work of plotting is the same, hence when this has been done it is as easy to draw a curve as to prepare Bars or fill in rectangles. In Diagram No. 8 the data appearing in the two previous diagrams have been plotted to the same scale as in No. 7, and the curve has taken the place of the bars. While this form of diagram is, at first, perhaps not quite so easy to understand as the previous one, a little practice will enable even an uneducated person to read it correctly. The rises and falls of the curve tell their own story, and anyone would be able to see *at a glance* exactly what the position was. This rapidity in grasping the significance of the data presented is one of the most important advantages the graphic method possesses over tabulation, and should prove of great benefit to a busy man desirous of being kept in touch with the details of his business. The time taken in preparing the graph is no more than that spent on a tabulated statement, and, moreover, it can be prepared by anyone, provided that rules are laid down for the purpose of guidance. As a matter of fact, in a business house the Charts would be standardised wherever possible after the utility of the various types had been tested, and from that point onwards the only necessity would be to mark the data on such standardised forms, and draw the curve, and

## DIAGRAM Nº 8.

Diagram showing ANNUAL TURNOVER for TWENTY YEARS.

i On Natural Scale.



this calls for no great skill. Another important advantage of the curve is that it shows the data as a continuous line, and thus is continuous in its effect, whereas by the Bar method the data for a particular year is shown as a separate entity, which in the case of a business is hardly to be conceived. This continuity of the diagram gives the business man valuable information as to the progress or otherwise of his business.

It will be apparent from Diagram No. 8 that the Curve takes up but little of the space available for the purposes of the diagram, so that plenty of room remains for the plotting of factors which relate to or affect the data shown in the principal curve. In the diagram under review the ten-year moving average as shown in column (*d*) of Table G has also been plotted to show the Trend, and it becomes at once apparent that heavy though the various falls have been the elimination or reduction of these short-term fluctuations has had little or no permanent effect upon the progress of the business, for the moving average line moves steadily upward throughout the whole of the period, and this upward trend is particularly noticeable in the last four years. It should be particularly noted that the moving average is so plotted that each point falls in the middle of the period from the figures of which the average is calculated. This method of plotting an average should always be adopted, for by this means it is possible to discern the periods covered by the computation in an easier manner than would be the case were the points plotted to coincide with the final year of the period.

### (1) The Natural Scale Graph.

This method of plotting the actual numbers or results is known as the **Natural Scale Method,** and though it has very many valuable advantages, it suffers from the very serious disadvantage that we are obliged to show rises and falls of equal magnitude by the same vertical distance Thus, if the turnover in any one year happens to be £2,000, and in the next period £4,000, we have an increase of £2,000, and the curve would move up the vertical scale exactly the same distance as when the turnover increased from £100,000 to £102,000, since the actual increase in each case is the same, though in the first case the increase is 100 per cent., while in the second case it is only 2 per cent. of the previous year's figures. If the turnover in the second case had increased from £100,000 to £200,000, the line would have risen fifty times the height of the rise shown in the first case, although the ratio of increase was exactly the same, and therefore, unless care is exercised in the reading of the graph, fallacious conclusions as to the progress may be drawn. It is apparent, therefore, that if percentages were plotted we should perceive the ratio which the rises or falls bore to the base year, which would, of course, be equal to 100 per cent. This, again, is a disadvantage, since we frequently desire to know what ratios the fluctuations bear to previous periods, and, moreover, the work of calculating percentages frequently allows errors more or less serious to creep in.

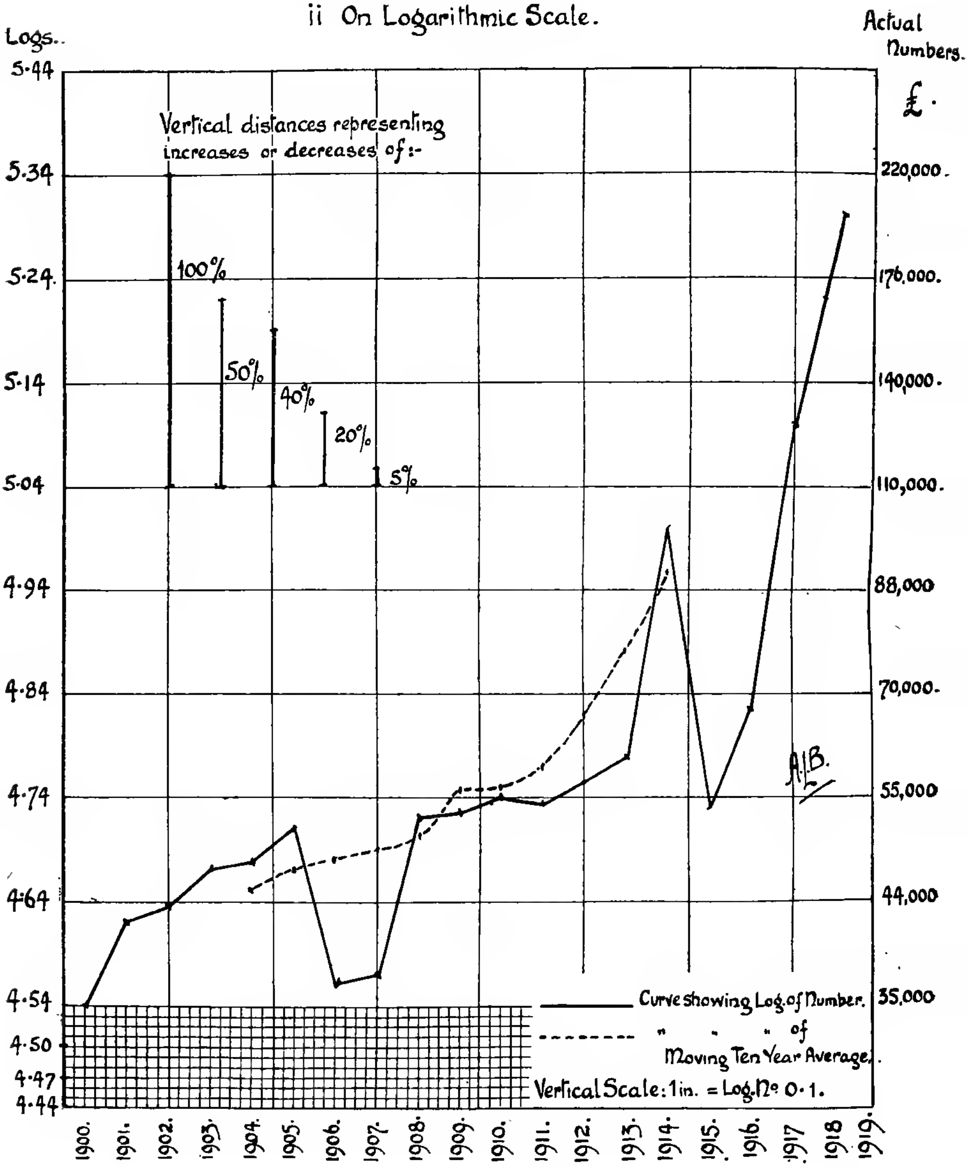
### (2) THE LOGARITHMIC SCALE GRAPH.

When it becomes advisable or necessary to consider the ratios of increase or fall, some other method of graphic presentment becomes absolutely necessary unless the whole utility of the method is to be destroyed. Such presentation of ratios may, however, be carried out by means of a graph plotted on a **Logarithmic Scale,** and thus the advantages which the graphic method enjoys over the tabulated statement may be continued. If we examine Diagram No. 8 it appears that the increases in turnover which took place in 1908 and 1916 are almost equal in their rises on the vertical scale. As a matter of fact, however, the increase in 1908 was 41·3 per cent. on the previous year's turnover, whereas the increase in 1916 was only 23·15 per cent. of the preceding year's figures, a difference that Diagram No. 8 fails to disclose. If we now turn to Diagram No. 9 we see that the difference in the ratio of increase is well defined, the rise in the curve for the earlier period being greater than that of the latter. This last-mentioned diagram has been constructed on a Logarithmic Scale, on which the Logarithms of the actual numbers have been used instead of the real numbers themselves. As is well known, if we add the Logarithms of two numbers we are actually multiplying the numbers themselves, and by converting the Logarithmic answer to the natural numbers we obtain the product. This is based on the Algebraic formula that when multiplying two expressions we add the indices, thus $x^2 \times x^3 = x^5$, a result obtained by adding the indices of the base $x$. Logarithms are really the indices of a defined base figure (usually 10), and hence adding the Logarithms of two numbers is equivalent to adding the indices, and thus to multiplying the actual numbers themselves. To construct a Logarithmic Scale we first find the Logarithms of the numbers we desire to plot and divide our scale into such a number of equal divisions as will allow all the Logarithmic numbers to be included, the Logarithmic numbers progressing in a uniform manner. Thus in Diagram No. 9, 1 inch on the vertical scale has been made to equal Logarithmic number 0·1, so that each smaller division will be equal to Logarithmic number 0·01. The Logarithms of the actual numbers are then plotted instead of the numbers themselves, with a resultant curve as shown. If we add the Logarithmic Number 0·301 to the Logarithm of any number we double the number. Thus the Logarithm for the number 20 is 1·3010. Add to this the Logarithmic number 0·301, and we get 1·6020, which converted into the natural number gives us 40, or twice the original number, so that in our

## DIAGRAM No 9.

Curve showing ANNUAL TURNOVER for TWENTY YEARS.

ii On Logarithmic Scale.



scale a rise of 3 inches will mean an increase of 100 per cent., and other rises in proportion. *The same result is obtained at any portion of the scale.* It must, however, be carefully remembered that in plotting the data on a Logarithmic Scale we are plotting ratios of rise and fall, and not the rises and falls themselves. A careful comparison of Diagrams 8 and 9 will reveal the fact that the fall in the Turnover which occurred in 1906 is emphasised in the latter chart as being more serious than would appear in the former, while the final advances which took place in 1918 and 1919 do not appear so large owing to the fact that though the actual figures increased considerably the ratios did not move in the same degree. A diagram prepared on a Logarithmic Scale is very useful just so long as it is remembered that the movements of the curves indicate ratios only. If, however, diagrams are prepared on both the Natural and Logarithmic Scales much useful information can be obtained by a comparison of the two. No special skill is required to construct a Logarithmic Scale Curve beyond the elementary knowledge necessary to find the Logarithmic Number of a Natural Number by the aid of one of the numerous Logarithmic Tables obtainable. The value of the Chart may be increased if the natural numbers are shown on a separate scale on the right-hand side of the diagram, as from it an approximation of the actual numbers represented may be obtained; but care must be taken to rely only upon those of the larger divisions unless conversion be made of the Logarithmic number of any other point for which it is desired to obtain the actual figures of the data. Bars should be shown on the chart to indicate the ratio of increase, and these bars are very useful for measuring the actual ratio of rises and falls in the curve.

Whenever the Natural Scale is being used, the base line of the diagram should always represent zero, and if this is impracticable by reason of the fact that by so doing the

Chart will be too large to be conveniently handled, its position should be indicated so that no erroneous conclusions will be drawn as a result of the true perspective of the increases and decreases not being apparent. This is a very serious error of many of the diagrams now being presented to the public. In using the Logarithmic Scale, however, no base line should be inserted, or fallacious conclusions may be drawn from the diagram. Another great advantage of the Logarithmic Scale is that the curves may be placed at any part of the diagram which is most convenient, since the ratios are the same at whatever point the curve is plotted, and thus two curves may be brought into close proximity for the purposes of comparison. This advantage is denied the natural scale, for if the scales on which the data is plotted are varied the true relationship is not disclosed.

There are other methods of Graphic Presentation which lend themselves particularly to the display of certain types of data, but great care must be exercised in their use, and they frequently entail more work in preparation than does the graph proper.

### (d) The "Bar" Diagram to show Components.

**The Bar Method** of presentation is a popular one, and is particularly useful when one is desirous of showing the components of which a total is composed. The following Table shows the total amount of cheques "cleared" through the London Bankers' Clearing House *in millions of pounds sterling* during the years 1916 and 1917, and also how the total is made up, divided into the three clearings, known respectively as the Town, Metropolitan and Country Clearings.

| Year. | Town. | Metropolitan. | Country. | Total. |
|---|---|---|---|---|
| 1916 | £12,329 | £1,074 | £1,872 | £15,275 |
| 1917 | £15,700 | £1,177 | £2,244 | £19,121 |

In Diagram No. 10 we see these results reproduced in the form of a Bar diagram. This method has much to recommend it, for it is clear, easily understood, and, moreover, shows very definitely the changes which take place in the components of the Total. The Bars should always be constructed so that it is clear that only the linear dimensions are being compared, for if they are made too broad the proportion appears to have been considerably altered, a fact which may be proved by experiment. Any figures which may be appended should appear on the left of the diagram as in the specimen, as by doing this nothing is allowed to interfere with the varying ends of the bars themselves, for if figures are placed after the bars there is a tendency for the eye to compare the spaces occupied up to the ends of the figures, instead of to the ends of the bars, thus leading to wrong conclusions. When constructing a number of bars for purposes of comparison care must be taken to plot the components in the same order in each case, otherwise the utility of the diagram will be diminished if not actually destroyed. This kind of diagram may be distinguished from the Graphic method proper by describing it as the **Pictorial Method,** and though it lends itself admirably to comparisons when only one factor capable of being represented by linear measurements is concerned, care must be exercised if other factors necessitate the use of diagrams introducing area or volume.

### (e) Comparisons of Areas and Volumes.

When plotting diagrams for the purposes of comparison involving area it must be borne in mind that *the dimensions must vary as the square root of the area,* and not on any other scale. If, for instance, we present certain data in the form of squares, the second set of data being twice the size of the first, then if the area of the first figure be 1 square inch, the second will need to be 2 square inches in area, and the

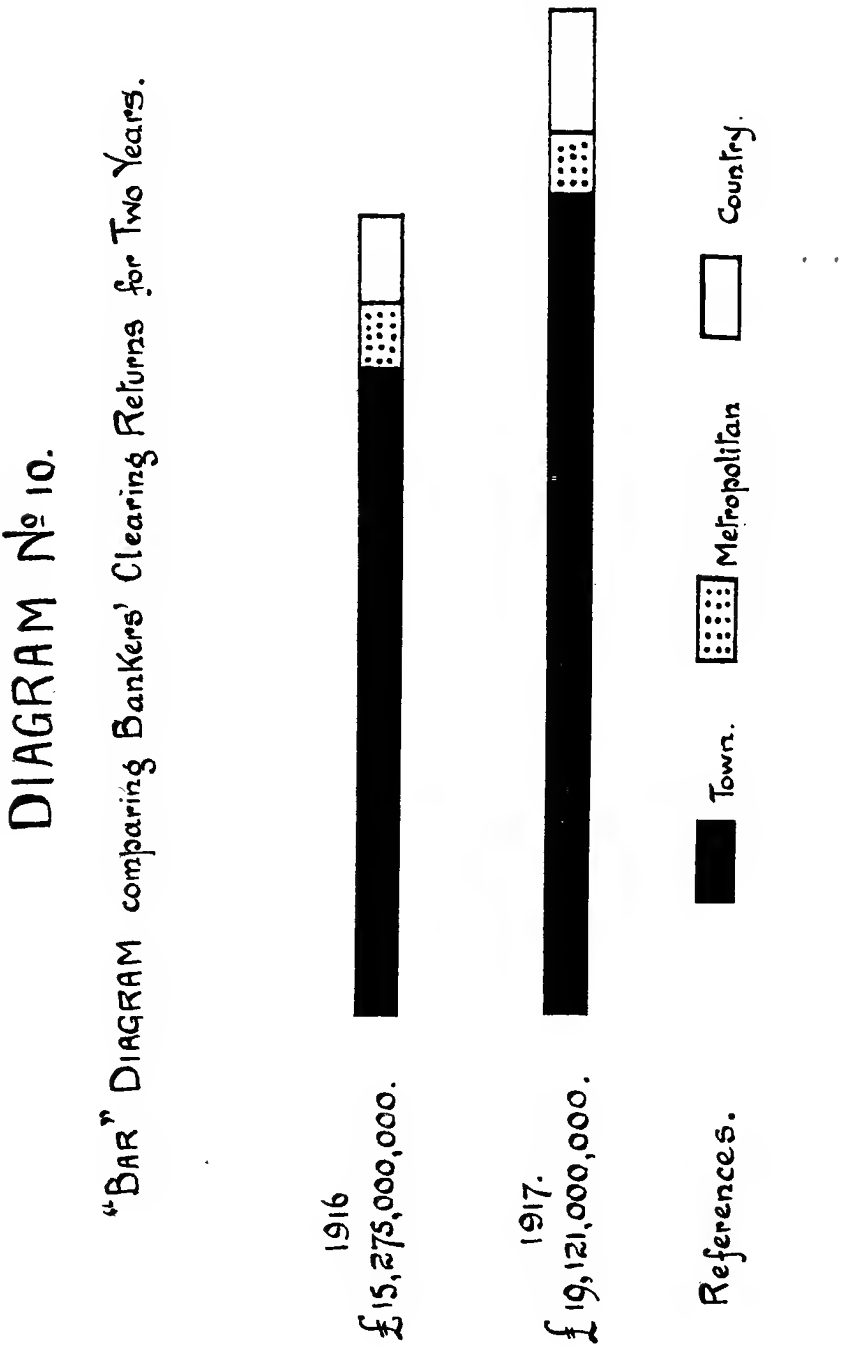


DIAGRAM № 10.

"BAR" DIAGRAM comparing Bankers' Clearing Returns for Two Years.

length of the base necessary to construct the figure can only be obtained by calculating the square root of the area, *i.e.* $\sqrt{2} = 1{\cdot}429$ inches. If the base of the first were, as it is, 1 inch, it would give a totally incorrect result to plot the second with a base of 2 inches, as by doing this the area of the second square would be 4 inches, and therefore four times the size of the first. In Diagram No. 11 the difference is clearly seen. Figure (*a*) is the original, and it is desired to show another figure twice the size of (*a*). This has been done in figure (*b*), whereas figure (*c*) has been constructed with a base twice the size of the first figure. The result of comparing figures (*a*) and (*c*) would be to give a fallacious idea of the true proportions. This same difficulty also appears when diagrams constructed in the form of circles are used, since here again we can construct circles for purposes of comparisons on the basis of area and on the basis of the diameter. Although the former is the more correct method of working, neither of the above gives very satisfactory results. In Diagram No. 12 figure (*b*) has been constructed with a diameter twice that of figure (*a*), while figure (*d*) has been drawn with an area twice that of (*c*), which coincides in all respects with figure (*a*). It appears that figure (*b*) is far more than twice the size of (*a*), while figure (*d*) does not appear to be large enough to be twice the size of (*a*). It is this fact that makes circles unsatisfactory wherever comparison is the object of the diagram. They can, however, be used with satisfactory results for diagrams showing the proportions which components bear to the whole. Diagram No. 13 shows such a circular diagram divided to show how the total of the London Bankers' Clearing Returns for 1917 is divided. The figures used are the same as in Diagram No. 10, and though it is very clear, and would be improved were the various divisions coloured, it cannot be used like the Bar method for comparison with other years. The best method to be adopted in the construction

# DIAGRAM № 11.

## Comparison of Squares on the Basis of Area and Base.

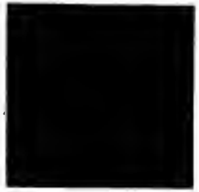


Fig. a.

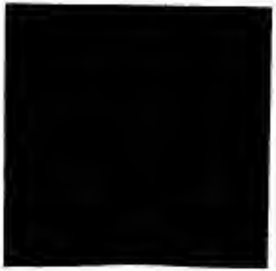


Fig. b.

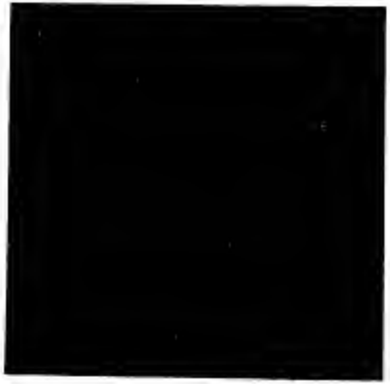


Fig. c.

# DIAGRAM № 12.

## Comparison of Circles on the Basis of Diameter and Area.

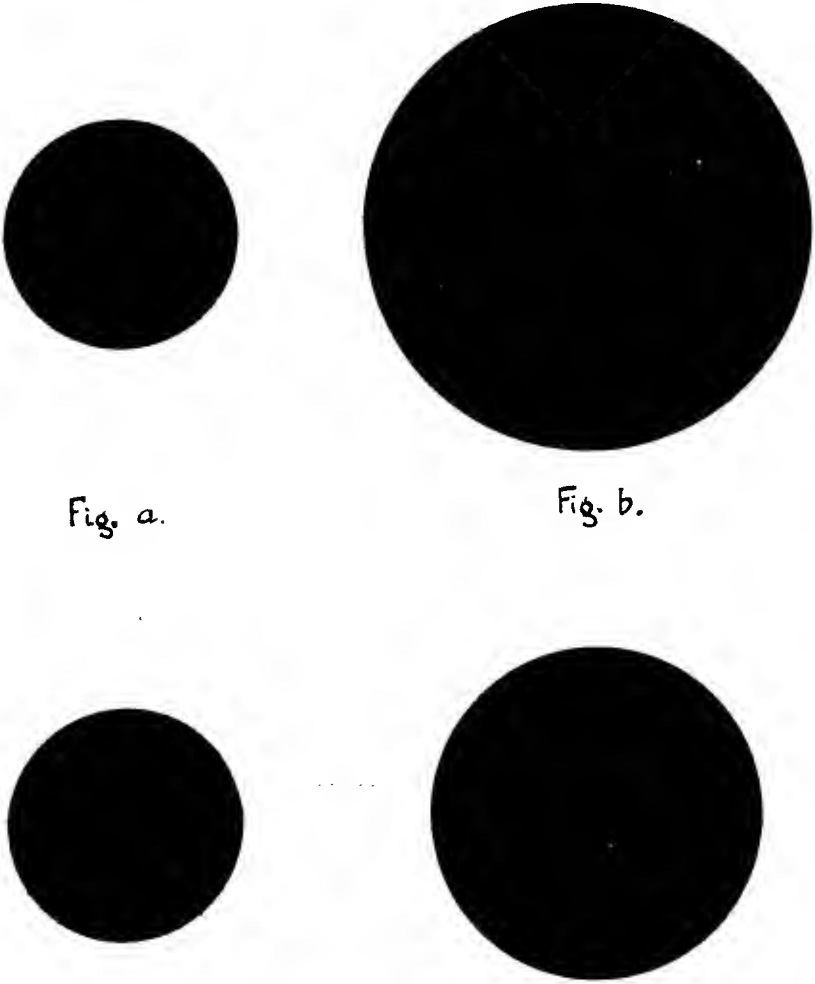


Fig. a.

Fig. b.

Fig. c.

Fig. d.

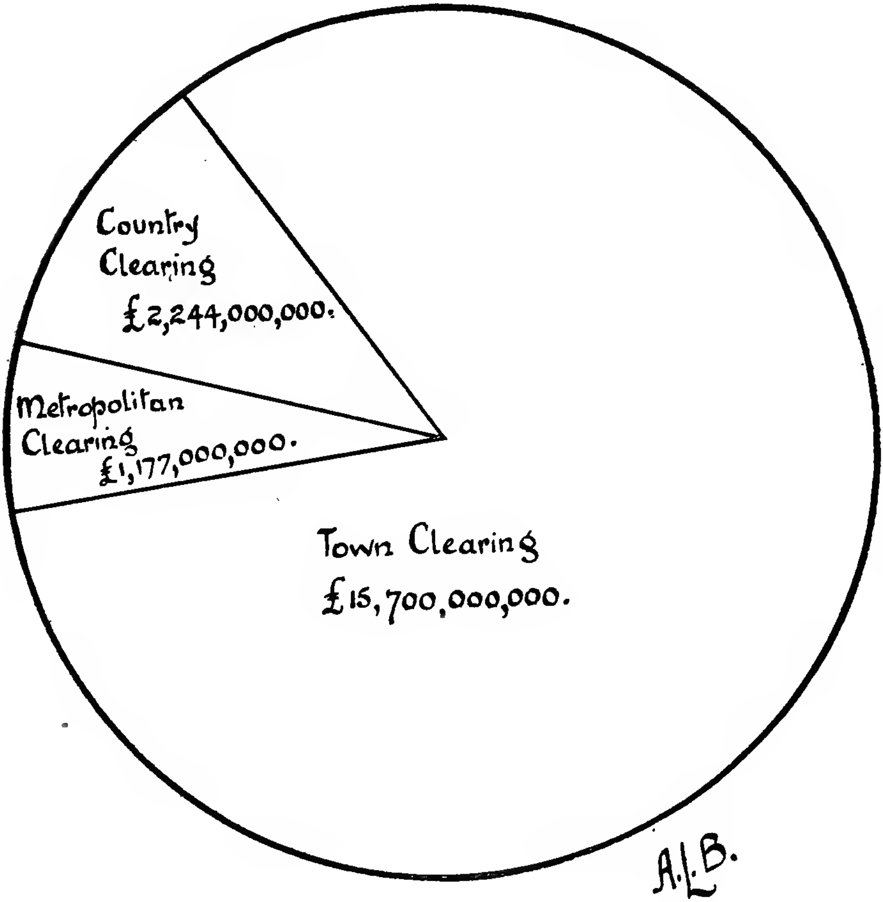
of such a circular diagram is to make a Table showing the ratios the components bear to one another, and then by ordinary proportional methods divide the number of degrees in a circle in the same proportions. Diagrams showing comparisons of volumes are never satisfactory, for it must be remembered *that the dimensions must vary as the cube root of the volume,* and the adjustment of the various dimensions calls for great care if the proportions are to remain unchanged, since if only one of the measurements be changed the change in the ratio is seldom very apparent.

### (f) Diagrams to show Three Factors.

Rectangular figures may occasionally be used to show three factors, such as is done in Diagram No. 14. Here the horizontal scale shows the profit made on each article sold, the vertical scale the number of the articles sold, and the area the amount expended on various items, and also the actual profit realised. In the case of figure (*a*) the profit per unit is seen to be £5, and therefore with the standing charges amounting to £60 for Salaries, £20 for Rent, £20 for Expenses, and £10 for Advertising, it is obvious that twenty-two units must be sold in order that such standing charges may be covered. Any article sold above this number will yield a clear profit, so that if thirty articles are sold in the period under review a profit of £40 is realised. If this is the capacity of the sales organisation—that is to say, if no more can be sold without an increase in the expenditure —then there would be no object in increasing sales unless the return was more than commensurate with the increased expenditure; but if the organisation is such that additional sales can be made without necessitating further expenditure, then it may pay the business to increase the sales up to the maximum obtainable on the fixed standing charges as shown, and the profit per unit sold might be reduced with the object of creating an extra demand as a result of the lower price.

# DIAGRAM No. 13.

Circular Diagram showing Bankers' Clearings for 1917

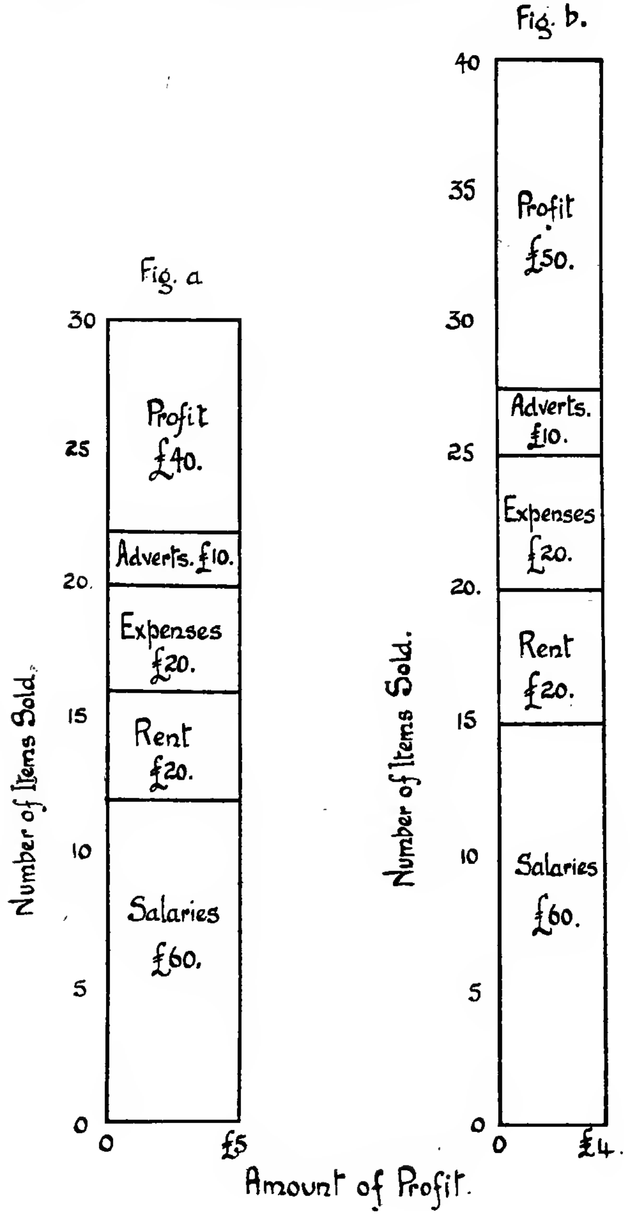


A.L.B.

TOTAL: £19,121,000,000.

## DIAGRAM No 14.

Illustration of Rectangles showing Three Different Factors.



If we assume the Sales capacity to be forty, then by reducing the profit per unit to £4, it will be necessary for twenty-eight to be sold before we clear our standing charges; but if we succeed in selling the whole of the allocation of forty units, we shall make a profit for the same period as before of £50, thus benefiting by the reduction in the profit per unit. Figure (*b*) shows the facts very clearly, and if such diagrams were prepared on the known facts, the management of a business might be able to judge policy more ably, to the advantage of all concerned. This particular method is not suitable for any other type of presentation, and great care must of necessity be exercised in its use.

Whenever it becomes necessary to show two scales on the same chart it is imperative that great care be exercised in the adjustment necessary to show the true relationship which exists between the two series of data being plotted and compared. If, for instance, the vertical scale on the one hand represents a greater number than does the other scale, the rises and falls of the curves will vary inversely as the scale, and hence the true perspective is not available for comparison.

## § 6.—**Rules for Constructing Diagrams.**

The following rules may be laid down to guide those desirous of obtaining the most satisfactory results from the use of the Graphic Method :—

1. The general arrangement of the diagram should be from left to right.
2. Where possible, represent quantities as linear magnitudes, since areas and volumes are more likely to be misinterpreted.
3. For a curve the vertical scale, wherever practicable, should be so selected that the zero line will appear on the diagram.

4. If the zero line of the vertical scale will not normally appear on the curve diagram, the zero line should be shown by a horizontal break, or by making the base line of the diagram broken. The former method is, however, preferable.
5. The zero lines of the scales for a curve should be sharply distinguished from the other co-ordinate lines.
6. For curves having a scale representing percentages it is desirable to emphasise in some distinct manner the 100 per cent. line, or other line used as the basis of comparison.
7. When the scale of a diagram has reference to dates, and the period represented is not a complete unit, it is better not to emphasise unduly the first and last ordinates, since such a diagram represents neither the beginning nor end of time. When the diagram is brought up to the previous year to that in which the diagram is constructed, the last ordinate should not be ruled in at all.
8. It is better that only those co-ordinate lines necessary to guide the eye when reading the diagram should be shown on the diagram.
9. It is advisable that the curve lines of the diagram should be very sharply ruled, as so to show the curves as distinct from other lines.
10. In curves representing a series of observations it is advisable, whenever possible, to indicate clearly on the diagram all the points representing the separate observations.
11. The horizontal scale for curves should usually read from left to right, and the vertical scale from the bottom to the top.
12. When curves are drawn on logarithmic co-ordinates the limiting lines of the diagram should each be at some power of ten on the logarithmic scale.

13. Figures for the scale of a diagram should be placed at the left and at the bottom, or along the respective axes.
14. It is often desirable to include in the diagram the numerical data or formulæ represented.
15. All lettering and figures on a diagram should be placed so as to be easily read from the base as the bottom, or from the right-hand edge as the bottom.
16. The title of a diagram should be made as clear and complete as possible. Sub-titles or descriptions should be added where necessary, in order to ensure clearness.
17. Where two or more curves are shown in the same diagram, they should be ruled in different coloured inks, or varying types of lines.

## SYNOPSIS TO CHAPTER VII.

# THE GRAPHIC METHOD APPLIED TO COMMERCE.

§ 1.—Records of Sales.
2.—Utility of the Moving Average Line.
3.—Comparison of Quantities and Values.
4.—Purchases and Sales.
5.—Price Fluctuations.
6.—Gross Profit and Expenses.
7.—Departmental Records.
8 —Turnover and Net Profit.
9.—Factory Records.
10.—Cost Records.
11.—Transport Records.

## CHAPTER VII.

# THE GRAPHIC METHOD APPLIED TO COMMERCE.

We have already seen that many advantages accrue from the use of the graphic method, the main ones being the ease of compilation and the readiness with which the graphs can be read and the trend and the fluctuations followed. Every business man finds it necessary to call from time to time for data relating to such things as Sales, Purchases, Stock, Expenses, Cash Balances, etc., and if these were presented to him in graphic form in such a manner that comparisons could be made between two or more periods or two or more correlated items, they would do much to save the time of a busy principal and leave him free to devote his time to other matters, instead of his having to spend a considerable time in analysing tabular statements. Many businesses would show better returns were data regularly presented to the heads of departments or firms, so that they could see the exact position, and take such steps as expediency dictated for maintaining sales or output, keeping down expenses and eliminating waste.

### § 1.—Records of Sales.

In a trading concern the principal point to be watched is the Sales. The following Table shows the Sales for each month of two successive years, called for the sake of convenience "This Year" and "Last Year," together with the accumulated sales month by month, so that the total turnover up to the end of any month can be obtained without trouble.

### TABLE R.

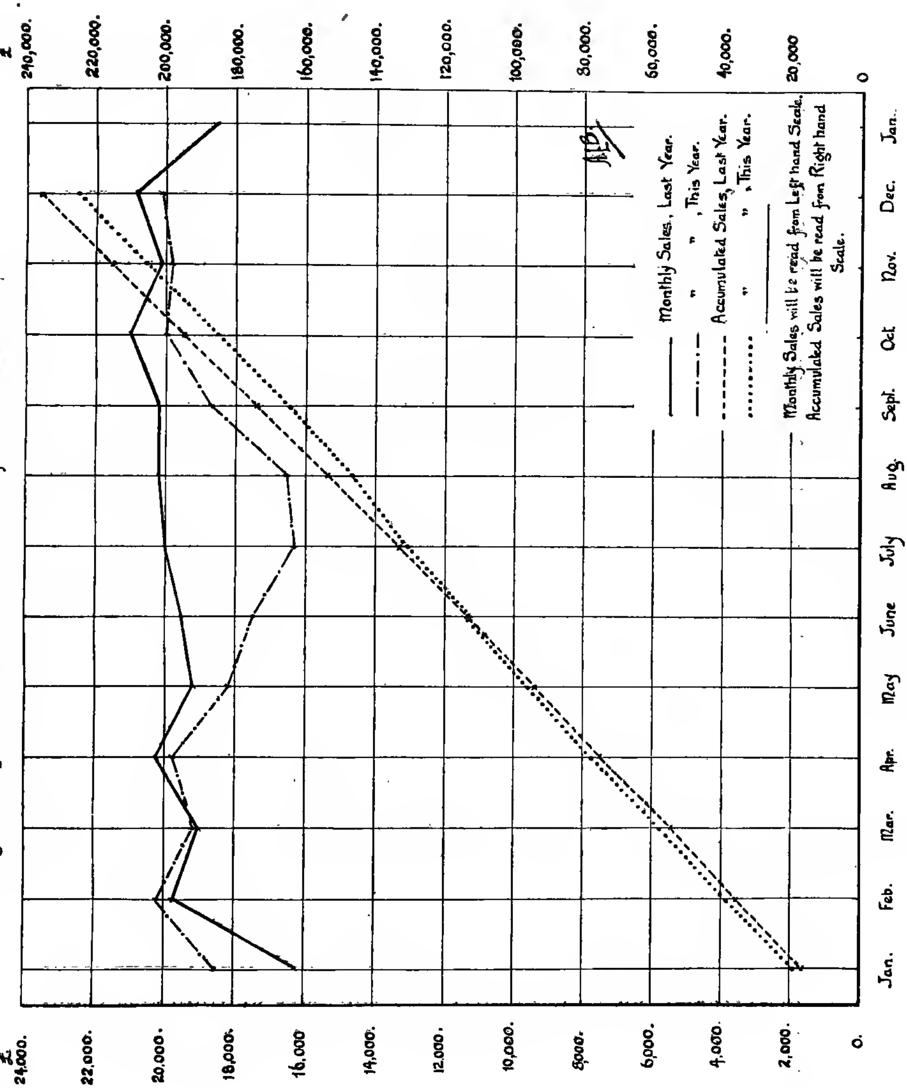
Table showing Monthly and Accumulated Sales for Each of Two Years.

| Last Year. | | Month. | This Year. | |
|---|---|---|---|---|
| Monthly Sales. | Accumulated Sales. | | Monthly Sales. | Accumulated Sales. |
| £ | £ | | £ | £ |
| 16,200 | 16,200 | January | 18,500 | 18,500 |
| 19,800 | 36,000 | February | 20,200 | 38,700 |
| 18,500 | 54,500 | March | 19,100 | 57,800 |
| 20,200 | 74,700 | April | 19,800 | 77,600 |
| 19,200 | 93,900 | May | 18,200 | 95,800 |
| 19,500 | 113,400 | June | 17,500 | 113,300 |
| 20,000 | 133,400 | July | 16,300 | 129,600 |
| 20,200 | 153,600 | August | 16,500 | 146,100 |
| 20,200 | 173,800 | September | 18,700 | 164,800 |
| 21,000 | 194,800 | October | 20,000 | 184,800 |
| 20,100 | 214,900 | November | 19,800 | 204,600 |
| 20,800 | 235,700 | December | 21,000 | 225,600 |

These data have been plotted in graphic form and are reproduced in Diagram No. 15. Clear though the Table may be to a trained mind, the graphic presentation of these figures is striking in the completeness and exactness with which the data are portrayed. In this diagram two different scales have been used, the left-hand one for use when it is desired to read the curves relating to the monthly sales, and the right-hand one (which is ten times the left-hand scale) when references have to be made to the accumulated sales graphs. By this means two distinct factors closely related to and dependent upon one another are capable of being read from the one chart, but as the data are distinctive and not correlated the curve for monthly sales cannot be compared with the accumulated sales. As in the case of the Table, the data would be recorded for the current year from month

DIAGRAM No 15.

Diagram showing Monthly and Accumulated Sales for Two Successive Years.



to month as the figures become available, and thus the chart would always be ready for instant reference, the latest available information being shown in its true perspective to the rest of the data. Comparison could thus be made month by month of the figures for the preceding period with those of the corresponding period of the previous year, the causes of divergence inquired into, and, when necessary, steps taken to rectify errors or faulty methods of carrying on the business revealed by such inquiries. The reason or reasons for any abnormal fluctuations should always be noted for future reference, as, for instance, where a special type of advertising campaign produces much better results at a certain season than at any other, as, if such facts are known, and the knowledge so obtained is applied when similar conditions are again present, sales may be maintained, or even increased, and thus the progress of the business may be assisted. Similarly, if the business be one in a holiday resort, weather may have a very serious effect upon the turnover, and information of this nature is necessary when only " Short Term " fluctuations are being considered, and, moreover, the data will indicate to the head of the business the extent to which it might be advisable to take out a policy of insurance against loss which might be occasioned by continuous bad weather. The lines showing the Accumulated Sales are also extremely useful, for if the total sales to date are " lagging " behind those of the previous year, inquiry would be caused, and thus steps could be taken where possible to increase sales, since the fall may be due to the goods purchased being unsuitable, insufficient advertising, lack of initiative on the part of the sales staff, price cutting by local or other competitors, or to a general slump in trade. Most of these can be rectified if only the head of the firm is made aware of what is wrong. The best method of presenting the position as far as the business is concerned is by the graph, for every fall is clearly shown, and no time is lost before making the necessary inquiries. Many busy men place tabulated matter aside till a more auspicious occasion for studying it presents itself, and when this examination is postponed the mistakes and faults continue, and the business suffers. The presentation of a graph to the principal attracts his attention to the salient points, and he can thus act without delay, stop leaks, and prevent waste going on to the detriment of the business as a whole. In actual practice each curve on a chart would be drawn in a different coloured ink, thus rendering each clearly distinguishable, though if the lines are not very numerous varying types of lines may be used advantageously, as has, of necessity, to be done in the diagrams used to illustrate the arguments advanced in this manual.

## § 2.—Utility of the Moving Average Line.

It would probably be advisable in a diagram of the nature illustrated in No. 15 to plot and draw a curve showing the average Sales for, say, a period of five years, or such other period as would cover a complete cycle of trade. Comparison of the current year's line with the curve showing such average would instantly give important information as to whether the progress of the business was being maintained, for, as has already been emphasised, either of two actual years' results may be abnormal, and thus comparison is rendered difficult or impossible for practical purposes, or for purposes of inquiry or reference. In Diagram No. 15 it will be seen that there is a remarkable drop in the sales in " This Year " from May onwards, and this may be due to a customary fall in the sales, or alternatively to the sales of the preceding year being abnormally high. If, however, the Moving Average Line were shown and a fall appears during the same period, the observed result is not unusual.

Curves to show the same data for other years can also be plotted on this Chart, provided that different coloured lines or distinctive types are used, but care must be taken not to overburden the diagram, as if this is done its legibility is destroyed and it becomes an involved mass of lines.

This particular type of diagram lends itself admirably to a large number of variations, as, for instance, when another line is added to show the amount of Sales Returns, or, what is better, since such a curve will normally appear at a much lower place on the vertical scale, a line showing the *Net Sales*. If this be done, the curve will be in close proximity to the line showing the total or gross sales, and thus the fluctuations in the returns can be easily compared with the gross sales. Though returns may be expected in all businesses, and vary greatly, yet if they become very heavy it is a sign that the goods do not suit the customers, or the method of manufacture is faulty, and thus expenses are being incurred unnecessarily, a state of affairs which is not in the best interests of the business. The graph will clearly indicate this, and thus the attention of the head of the business is drawn to the facts and the necessary action taken to remedy any such undesirable conditions.

Though such charts as that illustrated in Diagram No. 15 show the changes which take place monthly in the turnover, weekly, quarterly, half-yearly, or seasonal changes can be shown with equal facility, while the turnover of Departments or Branches, or the sales of individual representatives, may also be plotted in the same way. From such data carefully compiled over a series of years the business man can find the period of the year when his sales are likely to be heaviest, and thus arrange his purchases and deliveries at such times as the goods are likely to be required, and so make the turnover of goods more rapid, and obviate the necessity for maintaining heavy stocks in warehouses or stockrooms with the consequent risk of depreciation and damage, and, moreover, working capital is not locked up in stock which is only demanded at certain well-defined periods. If, in addition to compiling data of the financial aspect only, records were maintained of the types of goods sold, and also the quantities, further useful information could be obtained and utilised to the advantage of the business. A certain large London Store recently advertised that its sales records showed that the sales of a particular commodity were always highest in a particular week in the year, and that therefore during this period in the current year they had made special arrangements for the display and sale of such goods. This is an example of really scientific management, the result being that the business applies itself to meeting just that need of the general public which is of the greatest importance at the moment, to the obvious advantage of the public themselves, and to the benefit of the business in maintaining or increasing the turnover as well as in general reputation. By such means as this many of the chances of incorrect buying on the part of a business are eliminated, losses are thus minimised, and success has a better chance of being realised. All these data are capable of being shown by the graphic method, and thus the busy head of affairs may be kept in constant contact with the movements and position of business with the expenditure of a modicum of time and trouble.

## § 3.—**Comparison of Quantities and Values.**

If Statistical methods are to be completely successful in the commercial world, it is essential that records be kept not only of the monetary values, but also of the quantities of goods handled, for when any change takes place in the purchasing power of money (*i.e.* when there is a general change in the prices of commodities), the financial turnover will be affected, and wrong conclusions drawn therefrom. If, for instance, the purchasing power of money declines (*i.e.* the price of commodities increases) and the turnover of the business measured in £'s sterling remains the same, then the business will not be progressing, since the actual quantity of goods sold will be less. Conversely, any increase in the cash value of the sales will not necessarily mean that the turnover of goods has increased. In order to ascertain the true position it will be necessary to compare not only the amount received or charged in respect of the sales, but also the quantities sold, as without this latter information it will be impossible to say that the business has progressed, for though the figures in the financial statements may show an increase, the actual quantities sold may be less than for the previous year, and, after all, the true test of a business is the quantity of goods sold, since any variation in price is bound to affect the financial accounts. In Diagram No. 16 an endeavour has been made to bring the two factors together in one chart for a period of ten years. The following Table gives the Turnover in £'s sterling and in Tons:—

TABLE S.

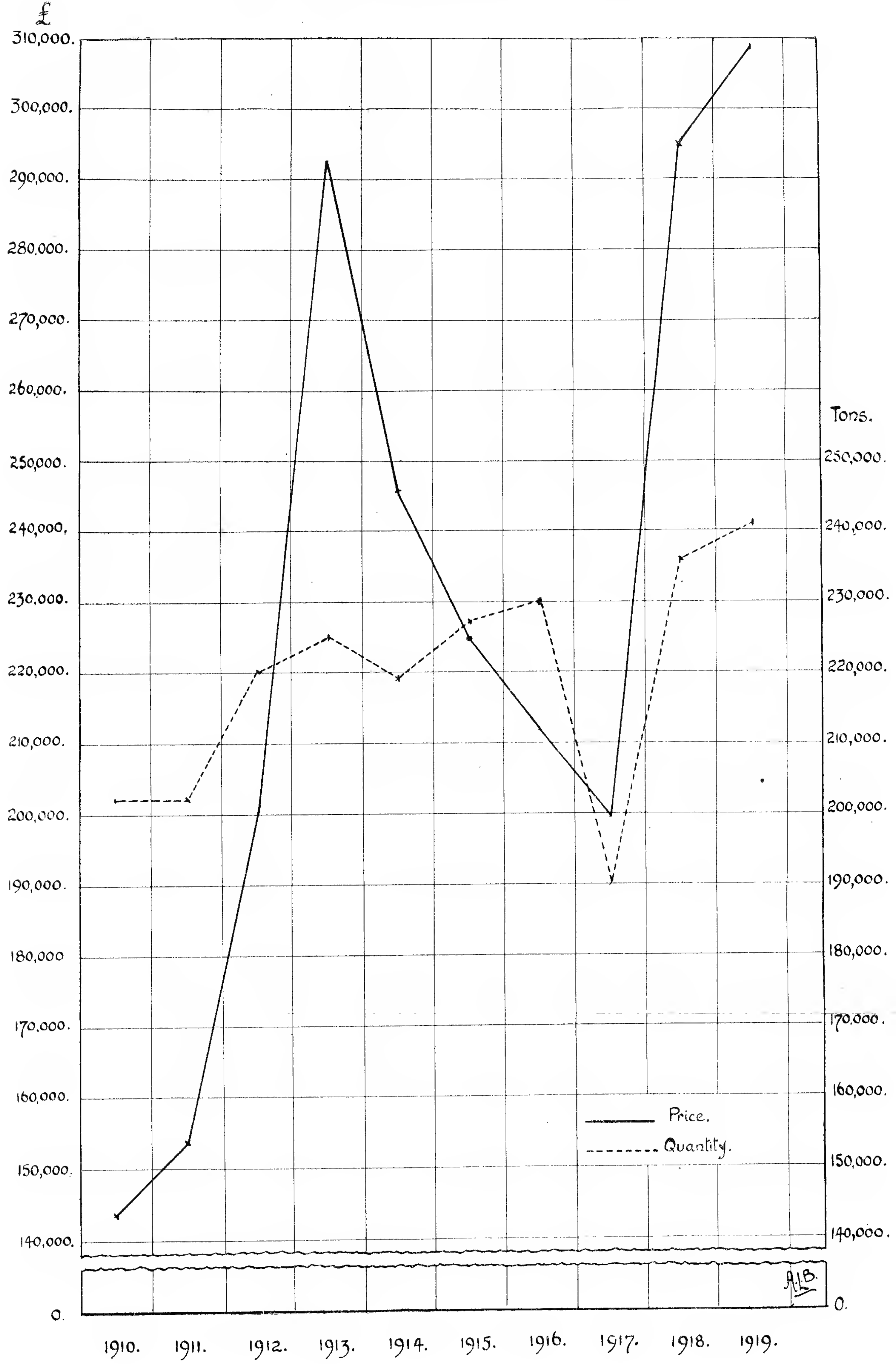
TABLE SHOWING TURNOVER IN £'S STERLING AND IN TONS FOR TEN YEARS.

| Year. | Turnover in £'s Sterling. | Turnover in Tons. |
|---|---|---|
| 1910 .. .. | 143,400 | 202,000 |
| 1911 .. .. | 153,500 | 202,000 |
| 1912 .. .. | 200,200 | 220,000 |
| 1913 .. .. | 292,500 | 225,000 |
| 1914 .. .. | 245,300 | 219,000 |
| 1915 .. .. | 224,700 | 227,000 |
| 1916 .. .. | 211,600 | 230,000 |
| 1917 .. .. | 199,500 | 190,000 |
| 1918 .. .. | 295,000 | 236,000 |
| 1919 .. .. | 308,400 | 241,000 |

In the diagram the monetary values will be read from the left-hand scale, and the quantities from that on the right-hand side of the chart. From the graphs thus plotted much valuable information can be obtained. For instance, the monetary value of the sales increased in 1911, although

# DIAGRAM No 16.

Diagram showing COMPARISON of QUANTITIES and VALUE.



the actual quantity of goods sold remained the same as in the previous year, while in 1913 the sales only increased from 220,000 tons to 225,000 tons, but the financial figures rose from £200,200 to £292,500. In the first of these cases we should have assumed that the business was progressing if we only considered the financial figures, while in the second we should get an entirely erroneous idea of the progress of the business were the quantities also not available for comparison. In each case the increase in the value of the goods sold is due to the fact that there has been a change in the price of the commodity. If the price were steady, we should expect to see the two curves fluctuating together. It will be seen from the diagram that the quantities sold tend to increase, and that in the ten years there was only one period which records a fall, *i.e.* 1917, and this fall was probably due to some effect of the War; but if we study the financial records we see that falls took place in 1914, 1915, 1916 and 1917.

Great care is necessary in plotting such a chart as this, for if a careful ratio is not prepared in connecting the two scales the real relationship between the two factors is destroyed or damaged. The unit of quantity should be plotted on a scale compiled after very careful study of the ratios existing between the unit of quantity and the monetary value. If, for instance, the records show that the price per unit of the given commodity averages, say, £5, then one unit of quantity and £5 can be taken as the basis of the scales, and each would be represented by the same vertical distance. If some such method be adopted the curves will be closely related, and hence comparison will be just, and the conclusions drawn therefrom will be reliable.

## § 4.—**Purchases and Sales.**

The relationship which exists between Purchases and Sales is very important, and it is a matter frequently overlooked by business men, with the result that a business often becomes overstocked, and thus valuable working capital is locked up and not available for more remunerative employment in some other part of the business. If therefore the purchases and sales of a business be plotted periodically on the lines shown in Diagram No. 17, it can be seen at once whether the purchases are too heavy, resulting in stocks accumulating unnecessarily, or whether they are not heavy enough, in which case the purchases will be insufficient to maintain the stock at that amount, which experience shows to be necessary for the well-being of the business. The following figures are those which have been plotted in Diagram No. 17:—

TABLE T.

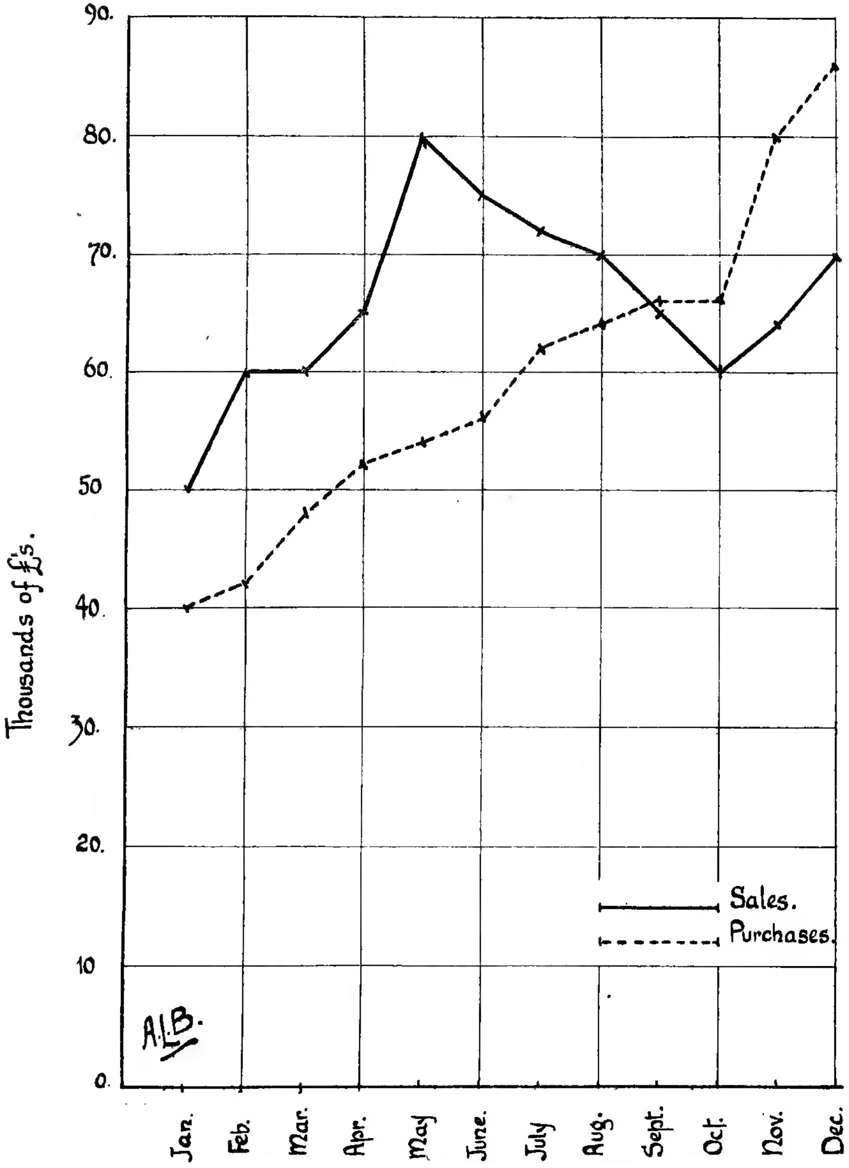
TABLE SHOWING MONTHLY PURCHASES AND SALES FOR ONE YEAR.

| Month. | Purchases. | Sales. |
|---|---|---|
| | £ | £ |
| January .. .. | 40,000 | 50,000 |
| February .. .. | 42,000 | 60,000 |
| March .. .. | 48,000 | 60,000 |
| April .. .. .. | 52,000 | 65,000 |
| May .. .. .. | 54,000 | 80,000 |
| June .. .. .. | 56,000 | 75,000 |
| July .. .. .. | 62,000 | 72,000 |
| August.. .. .. | 64,000 | 70,000 |
| September .. .. | 66,000 | 65,000 |
| October .. .. | 66,000 | 60,000 |
| November .. .. | 80,000 | 64,000 |
| December .. .. | 86,000 | 70,000 |

The diagram shows at once that up to May the purchases were much smaller than the sales, so that either old stock is being disposed of, or alternatively the stock is being gradually depleted. After the month of May the sales fall off, but the purchases line continues to advance, until in September more goods are being purchased than are being sold, that is, of course, from the point of view of

## DIAGRAM No 17.

Chart showing Combined Purchases and Sales for Twelve Months.



monetary value, so that expenditure on this one item alone will be in excess of the income. It may be argued that this is necessary to replenish stocks for the next period, which apparently is the period of rising sales, but it would appear to be poor policy to be buying steadily for eight months in the year in order to sell in the other four, though in certain types of business this procedure may be necessary.

## § 5.—**Price Fluctuations.**

If records of Sales are made over a number of years an average can be obtained, which if plotted for reference will give an interesting and reliable clue to the fluctuations of this item. Most businesses show a great many fluctuations in their returns, many of the fluctuations in, say, Sales, being well defined, and the periods in which they occur are regular or seasonal in their recurrence, and the average of a number of years will, as we have already seen, eliminate abnormalities, and we thus obtain a base upon which the quantities needed for any period can be estimated, and the necessary purchases made to meet the demand when it arises. While it is true that the period in which it is expected that the goods purchased will be disposed of may be, and sometimes is, abnormal, it is better to base our estimates and operations upon past known experience, and so eliminate some, at least, of the factors of chance which are present in all businesses, and which are uncontrollable. The chances of the purchases being too heavy for the sales of the period, with a consequent result that the stock carried becomes too large, can by such means as those mentioned be largely guarded against, though not entirely eliminated. If similar data be compiled as to the prices paid for the goods in which one deals, and the average over a series of years be taken, we shall find that with many of the commodities of everyday life prices tend to be higher at one season than at another, and knowing exactly when lower prices are to be expected arrangements can be made to purchase supplies at such times with a consequent reduction in the cost, thus enabling greater profit, or alternatively placing the buyer in a better position than his competitors who buy their supplies at times when the price is not so favourable. Again, the graphic method is by far the most suitable for presenting the facts of the case. In Table U (see next page) we see the average price of English Wheat calculated for a period of ten years. This average has been plotted for the sake of clearness in Diagram No. 18, and those periods when it is better to buy supplies are very clearly seen, so that an operator in wheat who could purchase his supplies in January would be enabled to make quite a good profit if he were able to obtain such a quantity as would enable him to meet his customers' demands for some months. The best periods in which to purchase wheat will be seen to be January, September and October, or thereabouts, and knowing this fact an operator could make the necessary financial arrangements to meet his needs. Importers and Exporters could compile similar records for the Rates of Exchange, and thus arrange their transactions so that payment would be made or received at such times as the exchanges were favourable to them. In this way goods could be purchased at a lower expenditure, and the purchaser be in a much better position than a competitor who did not pay any attention to the regularity with which many of the exchanges move. Clare in his manual on Foreign Exchanges shows by means of diagrams that when the rates of the exchange with America were averaged in this fashion the movements were very regular in their recurrence, and the same is at any rate true under normal conditions with all the gold exchanges, so that there is no reason why the ordinary business man should not reap some of the advantages to be obtained from the knowledge of the possibilities of these movements. While it is true that

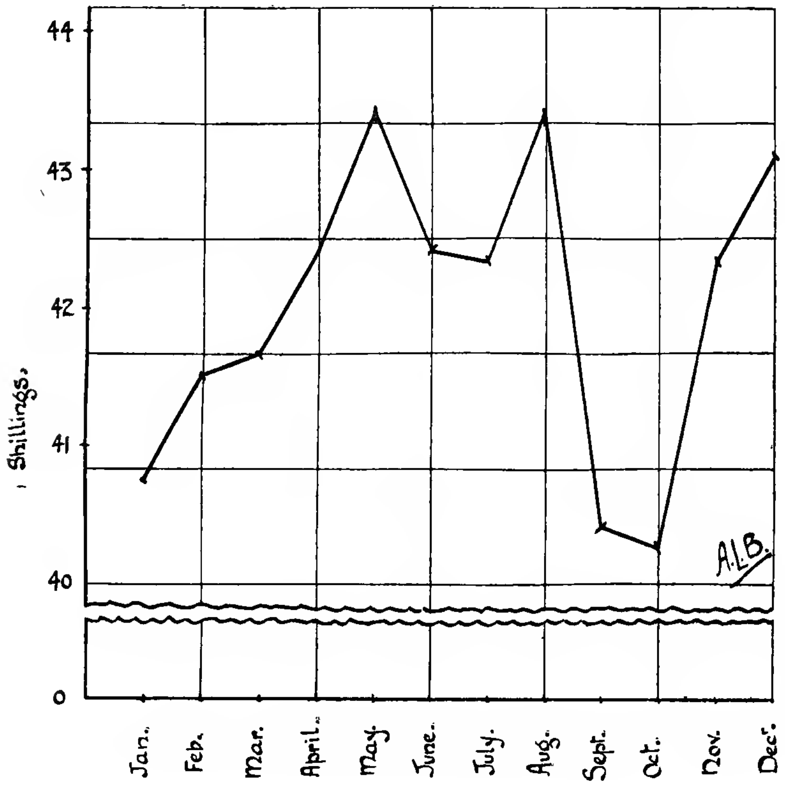
## TABLE U.

### Average Price of British Wheat per Imperial Quarter in England and Wales in Each Month for the Years 1908 to 1917.

| Year. | January. | | February. | | March. | | April. | | May. | | June. | | July. | | August. | | September. | | October. | | November. | | December. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | s. | d. | s. | d. | s. | d. | s. | d. | s. | d. | s. | d. | s. | d. | s. | d. | s. | d. | s. | d. | s. | d. | s. | d. |
| 1908 | 35 | 3 | 33 | 1 | 31 | 1 | 31 | 0 | 32 | 9 | 32 | 3 | 30 | 10 | 31 | 4 | 31 | 3 | 31 | 1 | 31 | 6 | 32 | 6 |
| 1909 | 32 | 8 | 33 | 10 | 35 | 6 | 38 | 5 | 41 | 11 | 42 | 9 | 43 | 3 | 42 | 4 | 34 | 7 | 31 | 8 | 32 | 7 | 33 | 2 |
| 1910 | 33 | 6 | 33 | 1 | 32 | 7 | 33 | 4 | 31 | 11 | 29 | 4 | 31 | 3 | 33 | 2 | 31 | 3 | 30 | 2 | 29 | 11 | 30 | 6 |
| 1911 | 30 | 8 | 30 | 4 | 30 | 1 | 30 | 5 | 32 | 0 | 32 | 2 | 32 | 2 | 31 | 8 | 32 | 0 | 32 | 9 | 33 | 2 | 32 | 10 |
| 1912 | 33 | 3 | 34 | 1 | 34 | 1 | 36 | 1 | 37 | 8 | 37 | 2 | 38 | 6 | 37 | 2 | 33 | 2 | 32 | 2 | 32 | 7 | 30 | 6 |
| 1913 | 30 | 6 | 30 | 11 | 31 | 1 | 31 | 5 | 32 | 7 | 32 | 8 | 33 | 5 | 33 | 8 | 31 | 8 | 30 | 11 | 30 | 3 | 31 | 1 |
| 1914 | 31 | 0 | 31 | 0 | 31 | 5 | 31 | 6 | 32 | 7 | 34 | 1 | 34 | 1 | 36 | 9 | 37 | 6 | 37 | 0 | 40 | 3 | 42 | 6 |
| 1915 | 48 | 8 | 54 | 11 | 54 | 7 | 55 | 3 | 60 | 10 | 57 | 5 | 52 | 2 | 54 | 2 | 43 | 6 | 46 | 4 | 52 | 11 | 53 | 9 |
| 1916 | 56 | 10 | 57 | 8 | 56 | 11 | 54 | 1 | 55 | 1 | 49 | 4 | 50 | 0 | 57 | 2 | 59 | 4 | 60 | 7 | 69 | 5 | 73 | 4 |
| 1917 | 75 | 9 | 76 | 2 | 79 | 2 | 83 | 10 | 77 | 10 | 78 | 1 | 78 | 2 | 77 | 11 | 71 | 0 | 70 | 9 | 70 | 3 | 70 | 9 |
| Average | **40** | **9** | **41** | **6** | **41** | **8** | **42** | **5** | **43** | **5** | **42** | **5** | **42** | **4** | **43** | **5** | **40** | **5** | **40** | **3** | **42** | **3** | **43** | **1** |

## DIAGRAM No 18.

Graph showing Ten Year Monthly Average Price of English Wheat per Imperial Quarter.



greater interest has been taken recently in the movements of the exchanges, when these again become normal the possibility of making additional profit by using the exchange fluctuations to the advantage of the business will probably be overlooked. Any " averages " so used would be adjusted from time to time, so that any tendency for the fluctuations to shift would be brought into account.

## § 6.—Gross Profit and Expenses.

There are many factors in Commerce which are so closely related to one another that consideration of any of them is best made when the others are also available for comparison and criticism. A good example of this is Gross Profit and Expenses. If we consider expenses only, then any increase may be the subject of needless criticism, since it may be discovered that the gross profit has increased also, and since the rate of gross profit is not likely to vary, there must have been a greater turnover, which may easily lead to greater expenditure; so that if it can be shown that the ratio of increase in gross profit is equal to, or greater than, the increase in expenditure, there will be little or no need for the expenditure to be adversely criticised. It must not, however, be assumed that there is no need to go into the question of expenditure, as this is a necessary procedure from time to time if real economy is to be effected; but if the standard of expenses is exceeded only by a lower ratio than that by which gross profit has been increased, then it is more than possible that the increase in expenses is due to the increase in the turnover, and to that only. If, however, the expenditure has increased and there has been no corresponding increase in gross profit, then it would appear that inquiry is necessary if the increase is to be justified, though it must always be borne in mind that the increase in sales may not always coincide with the increase in expenditure. This would happen when a special advertising campaign had been inaugurated, such as is necessary to place a new commodity upon the market, and therefore full inquiry needs to be made before conclusions can be drawn. It would be very useful to the business man to compare Gross Profit and Expenses and also the Net Profit, and this has been done in Diagram No. 19, which is plotted from the following data :—

TABLE V.

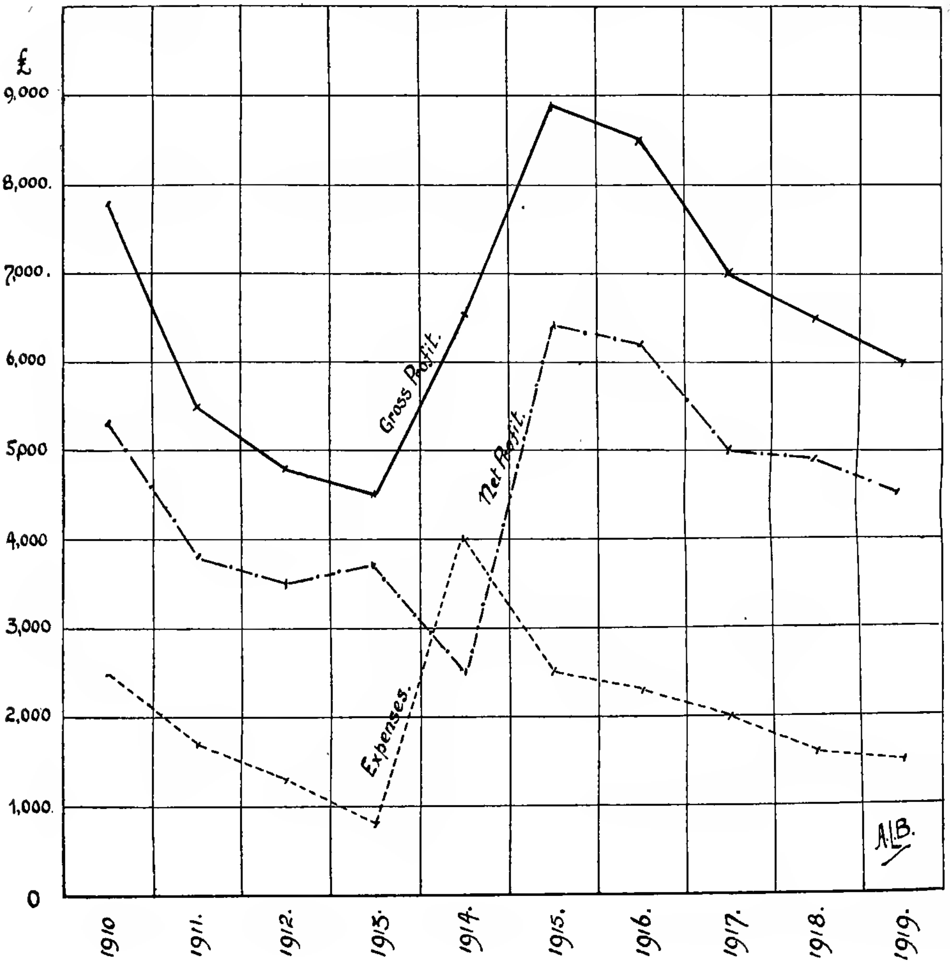
TABLE SHOWING EXPENSES, GROSS AND NET PROFIT OF A BUSINESS FOR TEN YEARS.

| Year. | Gross Profit. | Expenses. | Net Profit |
|---|---|---|---|
| | £ | £ | £ |
| 1910 .. .. | 7,800 | 2,500 | 5,300 |
| 1911 .. .. | 5,500 | 1,700 | 3,800 |
| 1912 .. .. | 4,800 | 1,300 | 3,500 |
| 1913 .. .. | 4,500 | 800 | 3,700 |
| 1914 .. .. | 6,500 | 4,000 | 2,500 |
| 1915 .. .. | 8,900 | 2,500 | 6,400 |
| 1916 .. .. | 8,500 | 2,300 | 6,200 |
| 1917 .. .. | 7,000 | 2,000 | 5,000 |
| 1918 .. .. | 6,500 | 1,600 | 4,900 |
| 1919 .. .. | 6,000 | 1,500 | 4,500 |

This statement presented to a business man would need careful study and analysis in order that the full significance of the figures could be appreciated, but the diagram shows immediately the divergence which exists. From the curves there plotted we can see that in the first three years the gross profit was falling, but the expenses were also falling, and consequently the net profit did not fall in the same ratio as the gross profit. This fall in the gross profit was in all probability due to a fall in the sales, as a reduction in the rate of gross profit is very unlikely, the percentage which is added to cost being a fairly constant factor. It is observable also that in 1912 the fall in the expenditure was accompanied by an even greater fall in the gross profit, and this constant falling in the gross

## DIAGRAM No. 19.

Diagram showing GROSS PROFIT, EXPENSES and NETT PROFIT for Ten Years.



profit earned while expenses are also on the downward grade would seem to point to the fact that economy in management was so great that it was seriously handicapping the business. This assumption is also borne out by the fact that in 1914 a sharp rise in the amount of expenses was accompanied by a considerable advance in the amount of the gross profit, though the latter advance was not in the same ratio as the former, with the result that the net profit in that year actually fell by £1,200. The advance in the gross profit was, however, even greater in the next year, though expenses actually fell considerably over those of the previous year, and from this it would appear that in 1914 some special effort was made to produce better results from the business, and the effect of this effort was also apparent in the year 1915. Subsequently the gross profit again declined, and this decrease was even more sharply apparent than the decline in the expenses. If one were investigating the business with a view to improving the results to be obtained therefrom, it would be necessary to call for explanations of the very heavy increase in the expenditure incurred in 1914, and it would be wise to analyse the expenses with a view to discovering the effect of each item upon the business. If it appeared that this heavy expenditure was due to an advertising campaign carried out during that year or some part of it, the whole of the cost of which was charged in that year's expenses, we should at once see that the decline in the gross and net profit was due to rigid economy adversely affecting the prospects of the concern, and we could then arrange that the "cheese-paring" policy should cease, and the business be placed upon a progressive footing by judicious advertising or other means. Here is an example of where the science of Statistics properly applied can assist a business, and lead to greater profits and expansion. Just as it is possible to be too lavish in expenditure, it is also possible to be too niggardly. By utilising Statistics in such a manner we are making it possible to trace faults, stop leaks, prevent waste, reduce the ratio of expenditure to turnover, or alternatively take steps to render the expenditure more remunerative—in short, to render our management more scientific, and so provide for contingencies whether possible or probable, thus eliminating or reducing the elements of chance which are present in all business or commercial undertakings.

## § 7.—**Departmental Records.**

There is no need for the graphic presentation of facts to be confined to the use of the office staff or the management. The efforts of the sales staff of any organisation might be spurred onward were they in a position to see how their departments or branches stand in relation to the remaining units of the organisation. If suitable Charts were prepared and exhibited to them showing the Departmental or Branch Sales day by day, week by week, or month by month, together with a curve showing previous results, or the results of some similar department or branch, they would be encouraged in their efforts to keep the line for current Sales from falling, thus assisting the development of *esprit de corps* and team work. One large London Store has already adopted some such idea, and reward that department which shows the greatest increase in its Turnover. Graphic methods of portraying the positions of the various departments are used, and the scheme has met with considerable success. By showing the various items of expense connected with the department or branch, and the way in which this compares with others, is also a good plan, as the tendency to waste is checked, and it would frequently pay the employers to pay a special bonus to the staff of any department showing a reduction in such items as Stationery, Wrapping and Packing Paper, etc. Charts could also be used to show the Cash and Credit Sales, and thus make adequate arrangements for safeguarding themselves against possible loss from Bad Debts. Receipts and Payments of cash might also be so recorded, and thus after a series of observations had been made it would be seen at what time of the year the greatest cash balance was accumulated, a very useful thing to know when the question of adequately financing a business is being considered. The results of an advertising campaign lend themselves admirably to treatment by the graphic method, and from them information can be obtained as to the best media for advertisements, the districts which yield the greatest amount of business, and even the best days or periods in which to advertise, all points of great importance if the business is to obtain the best results from its expenditure on advertising.

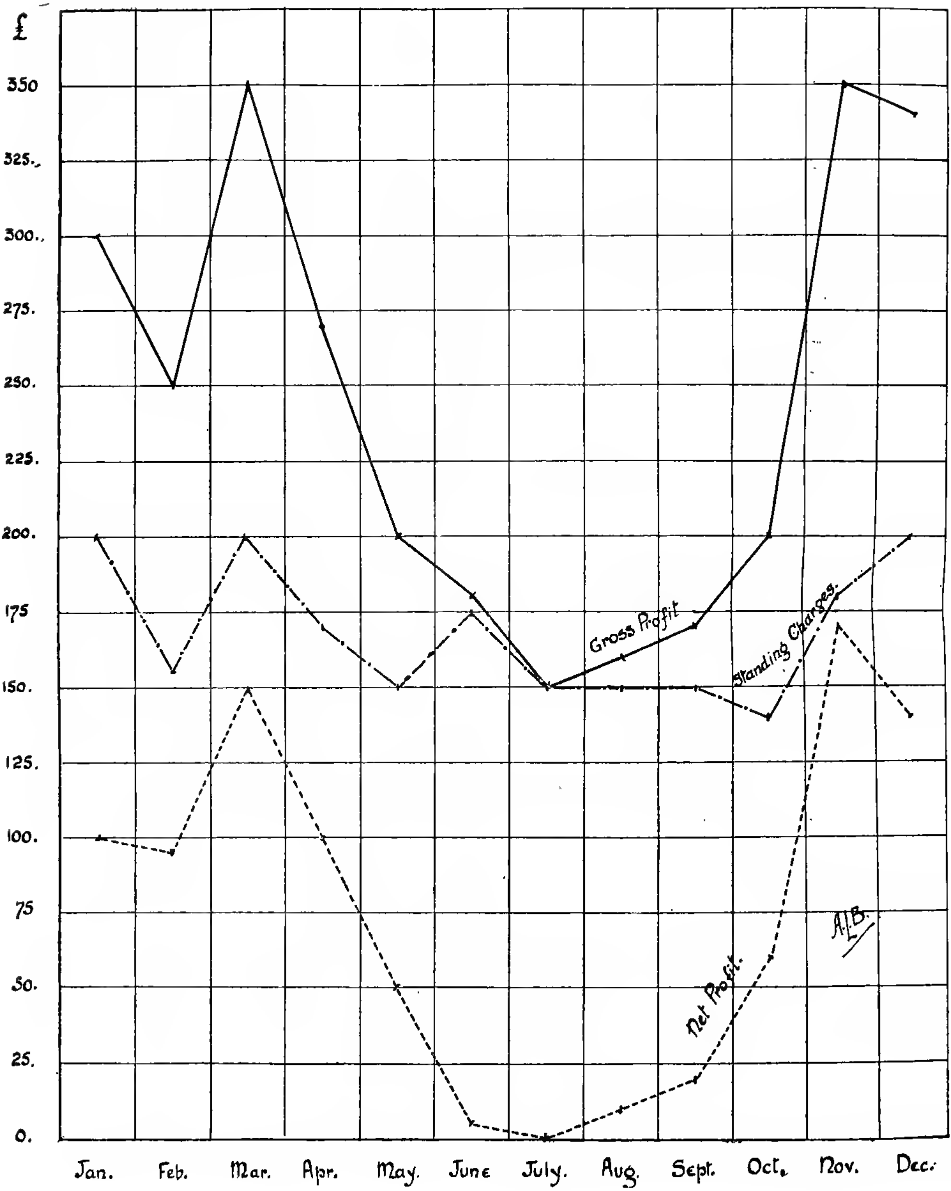
## § 8.—Turnover and Net Profit.

Where articles are being handled which are uniform in price and quality, the gross profit per article can easily be ascertained, and a chart prepared to show the Gross Profit on Sales, the Standing Charges which have to be met periodically, and the Net Profit made for the provision of depreciation, the payment of Debenture interest, dividends, etc., so that it will be possible to see at a glance if the sales are sufficient to yield an adequate return. Such a Chart is seen in Diagram No. 20, which has been prepared from the information given in Table W (p. 153). Each Article sold shows a gross profit of £1.

It will be seen from the diagram that the heavy fall in the gross profit due to the fall in the sales for the period subsequent to March was not accompanied by a corresponding fall in the standing charges, and consequently the net profit fell till in June there was a margin of £5 only, while in the next month the income from sales was only just sufficient to meet the current expenditure. Such a state of affairs might conceivably overtake any business,

## DIAGRAM No 20.

Diagram showing MONTHLY GROSS PROFIT, STANDING CHARGES and NET PROFIT.



particularly one of a seasonal nature, and if the facts are known, then provision can be made for curtailment of such expenditure as is unnecessary, or arrangements made to meet interest charges at some other time, or credit obtained to carry the business over the slack season. If this sequence of events were regularly recurrent, purchases could be made, so that payment fell due during the periods January to March, and October to December.

TABLE W.

Table showing Number of Articles Sold, Gross Profit, Standing Charges and Net Profit.

| Month. | No. of Articles Sold. | Gross Profit. | Standing Charges. | Net Profit. |
|---|---|---|---|---|
| | | £ | £ | £ |
| January .. .. | 300 | 300 | 200 | 100 |
| February .. .. | 250 | 250 | 155 | 95 |
| March .. .. | 350 | 350 | 200 | 150 |
| April .. .. .. | 270 | 270 | 170 | 100 |
| May .. .. .. | 200 | 200 | 150 | 50 |
| June .. .. .. | 180 | 180 | 175 | 5 |
| July .. .. .. | 150 | 150 | 150 | 0 |
| August .. .. | 160 | 160 | 150 | 10 |
| September .. .. | 170 | 170 | 150 | 20 |
| October .. .. | 200 | 200 | 140 | 60 |
| November .. .. | 350 | 350 | 180 | 170 |
| December .. .. | 340 | 340 | 200 | 140 |

## § 9.—Factory Records.

The graphic method can be used to advantage in the factory, and in many ways it lends itself admirably to the presentation of important data to the workpeople, who would probably not take any interest in tabulated statements. One factory producing a large number of similar articles of varying sizes charts the output of every machine, as shown by an attached automatic numbering machine, and places the chart in close proximity to the worker, so that he can see how his output is progressing day by day, and also compare it with the average output of all the similar machines employed on the same class of work for the previous year, which is also shown on the chart. The total output of each size of article is also plotted daily on a large chart in the office of the managing director, so that he is enabled to see at a glance whether the output is steady, increasing or decreasing, and ascertain the reasons for fluctuations. Any unusual circumstance is plainly shown on the chart, such as when a machine is not working, etc., and each type of article has its own distinctive coloured line. It has been found that the operatives take a keen interest in their charts, and endeavour to maintain a steady output, and therefore instantly report the slightest defect in their machines which might lead to any reduction of their output. The experience of the last few years shows that since the installation of this system the days lost by breakdowns to machines have almost disappeared, and the output has consequently increased from this source alone. The management can, by comparing the charts for various machines, which is done from time to time, discover those machines which are "lagging" behind the others, and ascertain the reasons therefor. Improvements have been made in the running of several machines as a result of this investigation. They are also enabled to gauge the relative merits of their operatives, and can reward the diligent and weed out the undesirables. The result is so marked, and the time taken in so recording the output so small, that it is a recognised part of the firm's organisation. The same firm also maintains charts showing the weekly percentage of time occupied by their employees in non-productive labour (such as cleaning machines and preparing them for the work in hand), and they can thus instantly see whether this item is tending to grow, and ascertain the reasons therefor. This chart has been found to be extremely useful, particularly when the

Machine-Hour Rate used in their costing system is under review or revision.

Another chart which could be used in a Producing Business would be one showing the Selling and Cost Price per article or per unit of production whatever that might be. This would be particularly useful in cases where the selling price fluctuates considerably, and is fixed by the operation of competition, for if the selling price does not show a sufficiently large margin over the cost, then the question of using the plant for some more remunerative work may well be considered.

## § 10.—Cost Records.

The Prime, Factory and Total Costs might be shown in one diagram with advantage. The PRIME COST of an article is the cost of the materials used plus the actual cost of the direct labour expended upon the material. The FACTORY COST is obtained by adding to the Prime Cost a proportion of the indirect charges of the Factory, such as Power, Unproductive Labour, Foremen's Wages, Works Managers' Salaries, etc., while the TOTAL COST is the Factory Cost plus the proportion of the Indirect Charges of Management generally, such as Office Expenses, etc. The indirect charges may be allocated to units of production in various ways, but frequently it will be found to be in the form of a percentage addition to the Cost. When this is the case the three Curves should move in complete agreement, but if some other basis is used the curves may be found to converge, and thus point to the fact that the Oncost is not being correctly allocated, or at least inquiry would need to be made to ascertain that the proportion added for these items is correct. A further chart which would be of great utility would show the ACTUAL expenditure in the works (other than that in respect of Material and Direct Labour), and the amount allocated to the contracts or output. These curves should practically agree. If the curve showing the allocation of the expenditure on whatever base it may be made falls below that showing the actual expenditure for the same period, the addition to the Prime Cost is insufficient and the Factory Cost used for further work is incorrect, since the actual expenditure will have been more than that added on to the cost of the work, with the result that the selling price will not show the gross profit anticipated, while if, on the other hand, the curve for allocations is above that of expenditure the additions made are too great, and consequently the Factory Cost is too high, with the ultimate result that any calculations based upon it will be incorrect, and the business handicapped when quoting a price against its competitors. A similar plan would be adopted with regard to the General Expenses and the allocation of the Oncost in arriving at the Total Cost; and the same considerations would apply to this case as to the former. When the business is working to contracts these results can in a large measure be checked, if considered necessary, by a graph showing the accumulated output for a given period with the actual total cost thereof, and also the contract prices obtained. If an inspection department is maintained in the works, Waste can be checked by plotting the percentage that faulty output bears to the total, and inquiring into the reason for any increase over what experience shows to be normal. Fluctuations in wages can be kept in view by a chart showing the number of workpeople employed, and the total wages paid, while output can be compared with the number of employees in the factory or works, or, better still, with the Man-Hours worked. This should enable steps to be taken to maintain a steady output.

## § 11.—Transport Records.

Where a firm is engaged in Road Transport on any scale, valuable information can be obtained by recording the number of miles each van or wagon travels, the number of hours employed, the weight of the load carried, etc., and this information should also be analysed to show the proportion that the miles travelled when empty bears to the total mileage covered. By such means steps can be taken to ensure that all similar vans or wagons carry similar loads and do not proceed half empty, or the rounds can be rearranged so that the number of miles run while empty can be considerably reduced. Similarly, by recording the costs of running, repairs, etc., the most economical vehicle can be found, and the personal skill of each driver indicated.

The graphic method is adaptable to any type of business or data, and consequently can be applied to a variety of commercial problems. A careful study of the object for which the data have been collected will enable the charts to be constructed to display this result with a clearness and ease unapproachable by any other method. The time taken is not greater, is indeed frequently much less, than that necessary to prepare tabulated statements, and, once the form of chart has been decided upon, it calls for no particular skill either in plotting or reading.

## SYNOPSIS TO CHAPTER VIII

# INDEX NUMBERS.

§ 1.—LOCATING CAUSES OF PRICE FLUCTUATIONS.
2.—PROPORTIONAL NUMBERS OR INDICES.
3.—CONSTRUCTION.
4.—INDEX NUMBERS IN USE.
5.—THE AVERAGE TO BE USED.
6.—MR. SAUERBECK'S COURSE OF AVERAGE PRICES.
7.—THE "STATIST'S" INDEX NUMBER.
8.—THE "ECONOMIST'S" INDEX NUMBER.
9.—THE BOARD OF TRADE INDEX NUMBER.
10.—INDEX NUMBER OF THE COURSE OF WAGES.
11.—USE OF INDEX NUMBERS FOR PURPOSES OF COMPARISON.
12.—IMPERFECTIONS OF INDEX NUMBERS.
13.—COST OF LIVING INDEX NUMBER.
14.—COMMERCIAL USES OF INDEX NUMBERS.
  (*a*) The Wages Problem.
  (*b*) Correlation of Factors.
  (*c*) Investment Index Number.
  (*d*) Costing.

## CHAPTER VIII.

# INDEX NUMBERS.

It frequently becomes necessary in Statistics to analyse movements, such as that of price, with a view to discovering the reasons actuating them. If the price of a commodity such as Wheat or Barley changes, the movement may be due to a large variety of causes, some of which may be summarised as follows :—

1. To a fall in production without a corresponding diminution in the demand ; hence a shortness of supply would occur, and following the economic law of supply and demand the price would increase.
2. To an increase in demand with no addition to the supply, in which case price would again increase.
3. To the reverse operation of either 1 or 2, with a resultant fall in price.
4. To a change in the purchasing power of money, for when the purchasing power of money increases the prices fall, while if the purchasing power decreases then the prices rise.

## § 1.—Locating Causes of Price Fluctuations.

It is frequently of as great advantage to the business man to know the reasons for the fluctuations of prices as for the statistican to record them and analyse their effects, for if the supply of a commodity is likely to be short, there is a field for the remunerative employment of spare capital; while if the demand is more than met or is falling off, it is useless from a commercial point of view launching a new venture to produce additional supplies of that particular commodity or article.

In the case of such commodities as are largely produced by nature, and the production of which needs the aid of natural forces, a shortage or an abundant supply can usually be anticipated some time ahead. In such a class of commodities would be all crops, though even the most conservative estimate of the possible production is likely to be wrong should some unforeseen calamity happen before the harvest can be gathered, such as a cloud-burst or floods. Weather may affect some of the crops adversely and not others, but when we find the price of all crop commodities moving in the same direction at the same time without any change being apparent in other commodities, there is strong presumptive evidence that the harvests have been good when the price falls, or have been bad when the price advances, and the greater the advance under such conditions the poorer the harvest. The location of such causes is a comparatively simple matter, but when we find the prices of commodities of a widely varying nature moving in the same direction at the same time the cause is not so apparent, and further inquiry is necessary if we are to find the true reason. If we find, for instance, that the prices of two commodities, such as Steel and Potatoes, are rising, it may well be that the change in prices is due to a change in the purchasing power of money, and that this purchasing power has fallen, since those factors which are likely to affect the price of potatoes are not likely to influence the price of steel. It might, of course, happen that a strike in the steel trade had shortened supplies at the same time that the crop of potatoes was short, and so caused prices of both commodities to rise. The best course of inquiry to pursue, however, is to eliminate any question of the influence of the changes in the purchasing power of money, and this is always likely to be present when prices of varying commodities change at the same time in the same direction, and, when the ratio of change is approximately the same, the evidence of this is much stronger. The following Table shows assumed average prices for these commodities for seven years, and will serve to illustrate the point aimed at :—

| Year. | Steel, per Ton. | Potatoes, per Cwt. |
|---|---|---|
| | £ | £ |
| 1913 | 7·5 | ·25 |
| 1914 | 7·0 | ·2375 |
| 1915 | 6·5 | ·2125 |
| 1916 | 7·0 | ·2375 |
| 1917 | 8·0 | ·2615 |
| 1918 | 9·5 | ·35 |
| 1919 | 10·5 | ·375 |

We cannot see from this Table whether the prices are subject to the same or similar fluctuations, and even when plotted on a chart, as has been done in Diagram No. 21, the position is no clearer, for though the price of steel shows very considerable fluctuations the curve showing the potato prices appears to be almost level. This is, of course, largely due to the difference which exists in the units used as a standard measurement of the two commodities. Had we taken a ton of potatoes the variations in the price of this commodity would have been more apparent, while they would have been still clearer had we taken 30 hundredweight of potatoes as being equal to a ton of steel, since it will be found that the price of a ton of steel is roughly thirty times that of a hundredweight of potatoes. In the diagram under review the variation of one-tenth of an inch on the vertical scale means a very small variation in the price of steel, but a very large fluctuation in the price of potatoes.

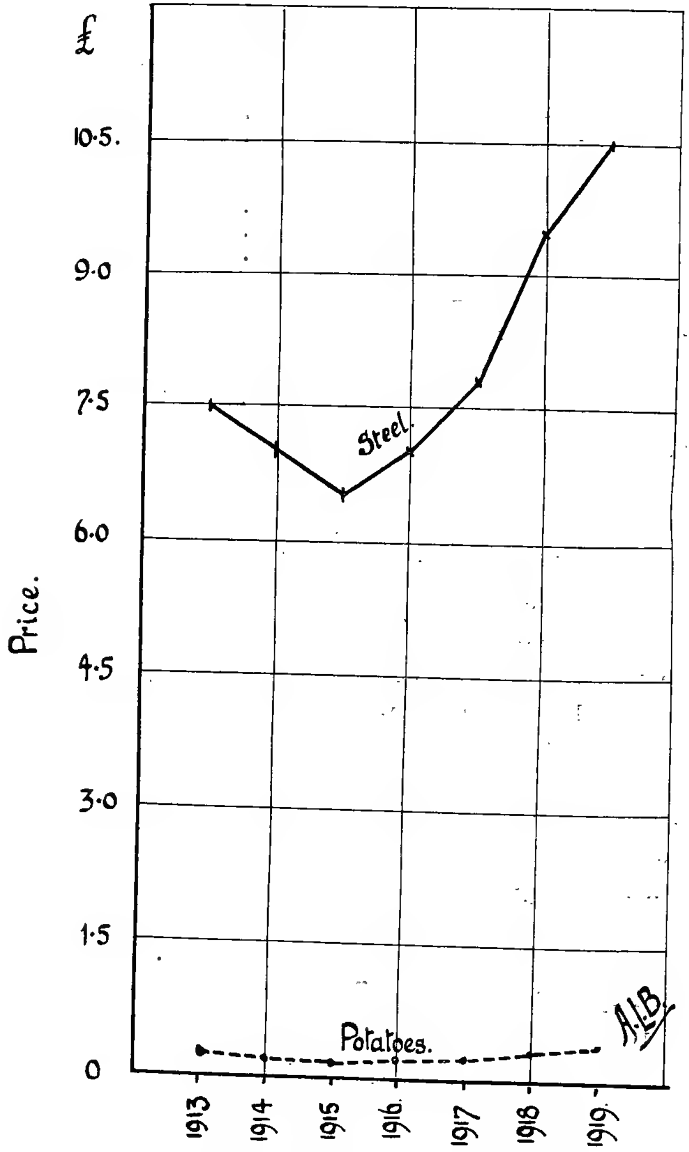
## § 2.—Proportional Numbers or Indices.

In order to compare the changes in price it will be necessary to use some method which will bring the two com-

## DIAGRAM № 21.

Chart showing the Fluctuations in the Prices of Steel and Potatoes.

i Actual Prices.



modities under nearly equal conditions, so that relative changes can be shown with ease. This can best be done by the use of **Indices,** using some well-defined data *of the same period in each case* as a base. This is usually done by taking the average price of some arbitrarily chosen period and equating this to 100, and then working the percentages which the price of each year bears to the price of the chosen base. An alternative method is to take the average of the years under review as the base, and this method is advantageous when the review of the data is to be carried out at the expiry of the time ; but if we are going to make the data of a continuous nature, the former method is preferable. If the average of the years in the Table be taken, the chance of abnormalities being present is reduced, for if we take an arbitrary period it may well happen that some factors are present which will affect the result. By following the latter method in this case we should find the result to be as follows :—

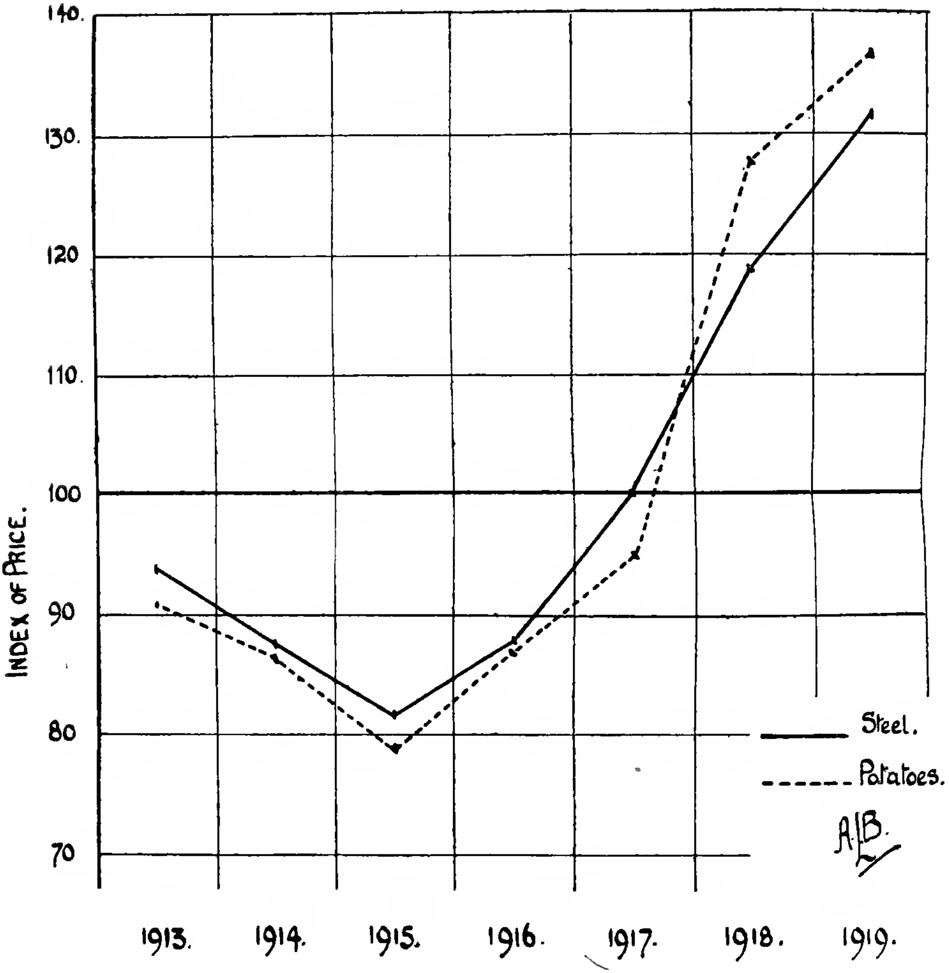
| Year. | Steel, per Ton. | Potatoes, per Cwt. | Index of Steel Price. | Index of Potato Price. |
|---|---|---|---|---|
| | £ | £ | | |
| 1913 .. .. | 7·5 | ·25 | 93·75 | 90·9 |
| 1914 .. .. | 7·0 | ·2375 | 87·5 | 86·36 |
| 1915 .. .. | 6·5 | ·2125 | 81·25 | 78·8 |
| 1916 .. .. | 7·0 | ·2375 | 87·5 | 86·36 |
| 1917 .. .. | 8·0 | ·2615 | 100·0 | 95·45 |
| 1918 .. .. | 9·5 | ·35 | 118·75 | 127·27 |
| 1919 .. .. | 10·5 | ·375 | 131·25 | 136·33 |
| Average .. | **8** | **·275** | **100** | **100** |

From this Table it will be seen that the fluctuations are very similar, but when they are plotted into curves, as has been done in Diagram No. 22, the relationship at once stands out clearly. It is true that there are minor discrepancies as in 1917, when the price of potatoes did not rise so sharply

DIAGRAM No 22.

Chart showing the Fluctuations of the Prices of Steel and Potatoes.

ii Indices.



as did the price of steel, and again in the following year, when the former rose more sharply than did the latter; but the general movement was obviously almost entirely the same, thus leading to the assumption that the same factor or factors was responsible for the change in price in both cases. What factor is likely to affect commodities so widely divergent in their nature and method of production in the same manner and to nearly the same degree? Practically the only common factor is a change in the purchasing power of money. The minor fluctuations observable in comparing the two curves can be accounted for by the fact that the potato crop is subject to natural forces which may vary in their effects, but which are not sufficiently powerful in their action to nullify the general tendency. Thus in 1917, though the price would advance as the result of the change in the purchasing power of money, the full effect has not been observable in this year owing to, say, a very prolific crop of potatoes, while in the following year the crop was a poor one, and this caused an extra advance in the price.

By means of such operations as this we are able to compare the relative changes in various commodities, which, when actual figures only are used, are difficult to bring together owing to variation in the basis upon which the price is fixed.

The effect which any change in the purchasing power of money has on commodities in general, and consequently on the business world as a whole, is a remarkable one, as has been increasingly recognised during the past two or three years. It is only by means of Indices that we are able to measure some of the changes which take place owing to the operation of this and other factors.

## § 3.—**Construction.**

The usual method of obtaining a general **Index Number** to measure the increases or decreases in the general level of prices is as follows: Certain commodities are selected whose *wholesale* price can be accurately ascertained or estimated, and the prices ruling for a series of years are tabulated. A base is then selected, either the price ruling in a particular year, or the average of a number of consecutive years (and this latter, as we have already seen, is preferable), and this base is made to equal 100, the same base period being used in each case, and each of the series of prices under review is then converted into percentages of the chosen base. We now have a series of proportionate numbers or Indices. To obtain the INDEX NUMBER for any period it is necessary to take the average of the Indices for that period. To illustrate the method let us consider the following data, which represent the Indices obtained by converting the wholesale prices into percentages of the average price of the base period, which in this case may be presumed to be the average price ruling during the years 1905 to 1909 :—

| Year. | Wheat. | Cotton. | Iron. | Wool. | Leather. | Silk. | Timber. |
|---|---|---|---|---|---|---|---|
| 1910 | 101 | 115 | 114 | 110 | 102 | 101 | 96 |
| 1911 | 97 | 110 | 120 | 145 | 98 | 103 | 98 |
| 1912 | 95 | 112 | 105 | 110 | 94 | 98 | 105 |
| 1913 | 103 | 114 | 90 | 112 | 96 | 96 | 99 |
| 1914 | 101 | 116 | 110 | 110 | 93 | 99 | 106 |
| 1915 | 115 | 110 | 160 | 105 | 110 | 120 | 120 |
| 1916 | 130 | 200 | 200 | 205 | 130 | 150 | 140 |
| 1917 | 180 | 310 | 240 | 315 | 180 | 200 | 200 |
| 1918 | 240 | 360 | 260 | 370 | 220 | 240 | 220 |
| 1919 | 310 | 500 | 320 | 510 | 310 | 290 | 320 |

## § 4.—**Index Numbers in use.**

In the actual computation of such an Index Number many articles in common use would be tabulated on the above lines, though frequently it is found more convenient to group them in some convenient manner, and the Index Numbers in common use vary in their methods and classification. Mr. Sauerbeck, in his Course of Average Prices, for instance, used forty-five common commodities, which are not divided into groups, while the *Economist's* Index Number is obtained after grouping forty-four selected commodities into five different groups, viz.: (1) Cereals; (2) Other Food Products (Tea, Sugar, etc.); (3) Textiles; (4) Minerals; (5) Miscellaneous (Rubber, Timber, etc.). The *Statist* uses the same commodities as did Mr. Sauerbeck, but divides them into two main groups, viz.: (1) Foodstuffs and (2) Materials. Each of these main groups is composed of three classes, the first named consisting of (*a*) Vegetable Foods, (Corn, etc.); (*b*) Animal Food (Meat and Butter); (*c*) Sugar, Coffee, Tea; while the second main class is composed of—(*a*) Minerals; (*b*) Textiles; and (*c*) Sundries.

## § 5.—The Average to be used.

To obtain the INDEX NUMBER for a particular year it is only necessary to take the average of the proportionate numbers (or Indices) for that particular period, but, as we are already aware, there are various types of averages which we could use. If we wish, for instance, to study the relative cost of living, it will be necessary to use an average entirely different from that which will give the desired result if we are investigating the effect of a change in the purchasing power of money. The purchasing power depends very largely upon the volume of money in existence and upon the speed with which it circulates. If we increase the volume of money in a country, or the speed with which it circulates, then, *all other things being equal,* we shall find the price of commodities in general will increase. If the increase in money is accompanied by an increase in purchasable commodities, then the rise will only take place in so far as the volume of money has increased beyond the volume of increased supplies. On the other hand, if the volume of money falls or its rapidity of circulation decreases, while the supply of commodities remains unchanged, then the prices will fall. For the purpose, then, of considering the effect of a change in the purchasing power of money, the rise in the price of any one article in common demand will be just as suitable as the rise in the price of any other similar commodity, provided, of course, that no extraneous circumstances have affected the price of either in addition to the alteration due to the change in the purchasing power of money. The presence of any extraneous influences can usually be discovered by a careful study of the Table of Indices or by tracing the divergence of curves when the Indices have been plotted on a graph, for if the proportional number of one commodity shows an abnormal fluctuation (as did, for instance, Wool in 1911, and Iron in 1915), which is not general to all or the majority of the commodities selected, then the presence of influences outside the change in the purchasing power of money is shown, and hence where possible such extraneous influences should be eliminated before endeavouring to obtain any indication of the trend of prices, that is to say, we shall endeavour only to consider the change which is brought about by one factor which affects the price of all the articles impartially. If we take the above Table of proportionate prices we shall find that the following will be the averages of the Indices we can obtain for each of the years dealt with :—

| Year. | Arithmetical Average. | Geometric Average. | Median. |
|---|---|---|---|
| 1910 | 105·5 | 105·3 | 102 |
| 1911 | 110·1 | 109·1 | 103 |
| 1912 | 102·7 | 102·5 | 105 |
| 1913 | 101·4 | 101·1 | 99 |
| 1914 | 105·0 | 104·7 | 106 |
| 1915 | 120·0 | 119·0 | 115 |
| 1916 | 165·0 | 161·8 | 150 |
| 1917 | 230·7 | 226·2 | 200 |
| 1918 | 272·8 | 266·7 | 240 |
| 1919 | 365·7 | 356·2 | 320 |

It will at once be seen that the three averages differ considerably, though the difference between the Arithmetic and Geometric Averages is so slight, especially in the earlier years, that the advantages accruing from the use of the Geometric Average are not sufficient to justify the large amount of extra labour involved in its calculation. As we know, the Arithmetic Average is affected by any extremes which may exist in the series, and this disadvantage is shared also, though to a smaller extent, by the Geometric Average. The effect of the extreme items is seen in the years 1911 and 1915, when the Arithmetic and Geometric Averages of the former are affected by the high index number of Wool, and of the latter by the high price of Iron; but its full effects are better seen in 1917, in which year the high numbers for Cotton and Wool greatly increase the average over the prices or numbers ruling in the majority of the items in the Table. It will be remembered that the Median is but little affected by the extremes, and thus in such cases as this, where all the proportionate numbers are not affected to the same degree, the Median will be found to be the most reliable guide to use, and this becomes increasingly so when the number of commodities under review is greatly increased. Not only so but it is certainly the easiest average to obtain, for it entails no calculation. It is obvious in the figures before us that the general tendency of the prices is to increase, as shown by an Index Number based on either of the averages we have extracted, thus indicating that the purchasing power of money has declined; but an examination of the indices upon which these averages are based shows that other factors were also present, and played an important part in the price changes, and therefore further inquiry would be necessary to locate and (if possible) measure the power of these factors.

By means of such an Index Number we are able to gauge any advance in the price of any one or more commodities, or of any group or class of article, and, moreover, by combining our various groups we obtain an Index Number for the whole of the commodities, and thus an indication of the general tendency, which is useful in so far that by increasing the number of commodities we tend to eliminate the effect of any abnormal fluctuations which exist in any one or more individual items. It will be realised that the principle underlying Index Numbers is one of "sampling," for no Index Number in existence includes all commodities, but only certain representative samples.

## § 6.—Mr. Sauerbeck's Course of Average Prices.

One of the most quoted Index Numbers is Mr. Sauerbeck's Course of Average Prices, and this is based upon the following commodities, the figures taken being indicated in the Table :—

1. Wheat, English. Shillings and pence per quarter.
2. Wheat, American. Shillings and pence per quarter.
3. Flour, Town made White. Shillings and pence per sack.
4. Barley, English. Shillings and pence per quarter.
5. Oats, English. Shillings and pence per quarter.
6. Maize, American Mixed. Shillings and pence per quarter.
7. Potatoes, Good English. Shillings and pence per ton.
8. Rice, Rangoon Cargoes. Shillings and pence per cwt.
9. Beef, Prime. By the carcase. Pence per 8 lbs.
10. Beef, Middling. By the carcase. Pence per 8 lbs.
11. Mutton, Prime. By the carcase. Pence per 8 lbs.
12. Mutton, Middling. By the carcase. Pence per 8 lbs.
13. Pork, average (large and small). Pence per 8 lbs.
14. Bacon, Waterford. Shillings and pence per cwt.
15. Butter, Friesland, finest. Shillings and pence per cwt.

16A. Sugar, West Indian Refining. Shillings and pence per cwt.  
16B. Sugar, Beet, German 88 p.c. f.o.b. Shillings and pence per cwt.  
17. Sugar, Java, floating cargoes. Shillings and pence per cwt.  
18A. Coffee, Ceylon plantation, low middling. Shillings and pence per cwt.  
18B. Coffee, Rio, good. Shillings and pence per cwt.  
19A. Tea, Congou, common. Pence per lb.  
19B. Tea, average import price. Pence per lb.  
20A. Iron, Scotch pig. Shillings and pence per ton.  
20B. Iron, Cleveland pig. Shillings and pence per ton.  
21. Iron Bars, common. £ per ton.  
22. Copper, Chili Bars. £ per ton.  
23. Tin, Straits. £ per ton.  
24. Lead, English Pig. £ per ton.  
25. Coals, Wallsend Hetton, in London. Shillings and pence per ton.  
26. Coals, average export price. Shillings and pence per ton.  
27. Cotton, middling American. Pence per lb.  
28. Cotton, fair Dhollerah. Pence per lb.

29A. Flax, St. Petersburg. £ per ton.
29B. Flax, Russian, average import price. £ per ton.
30A. Hemp, Manilla fair roping. £ per ton.
30B. Hemp, St. Petersburg, clean. £ per ton.
31. Jute, good medium. £ per ton.
32A. Wool, Merino, Pt. Philip, average fleece. Pence per lb.
32B. Wool, Merino, Adelaide, average grease. Pence per lb.
33. Wool, English, Lincoln half hogs. Pence per lb.
34. Silk, Tsatlee. Shillings and pence per lb.
35A. Hides, River Plate, dry. Pence per lb.
35B. Hides, River Plate, salted. Pence per lb.
35C. Hides, average import price. Pence per lb.
36A. Leather, Crop, hides. Pence per lb.
36B. Leather, average import price. Pence per lb.
37A. Tallow, St. Petersburg. Shillings and pence per cwt.
37B. Tallow, Town. Shillings and pence per cwt.
38. Oil, Palm. £ per ton.
39. Oil, Olive. £ per ton.
40A. Oil, Linseed. £ per ton.
40B. Seeds, Linseed. Shillings and pence per quarter.
41. Petroleum, refined. Pence per gallon.
42. Soda, crystals. Shillings and pence per ton.
43. Nitrate of Soda. Shillings and pence per ton.
44. Indigo, Bengal, good consuming. Shillings and pence per lb.
45A. Timber, hewn, average import price. Shillings and pence per load.
45B. Timber, sawn or split, average import price. Shillings and pence per load.

The base period selected for this Index Number was the eleven years 1867 to 1877, and to obtain the Index Number the arithmetic average of the Indices is taken. It will be seen from the above Table that the method of obtaining the prices upon which the Indices were based varied somewhat. In those cases where two or more varieties of a particular commodity were taken, as, for instance, Hides where three different types were listed, the arithmetic average of the various prices is the one used to obtain the proportionate number. Where the average import or export price has been taken, the procedure adopted is to take the Board of Trade Returns and divide the total value of the imports or exports of the particular commodity by the total number of units imported or exported as the case may be. The proportionate number of each commodity is calculated by simple proportion, the average price of the base period being equated to 100.

## § 7.—The "Statist's" Index Number.

The Index Number published by the *Statist* is based upon, and is really a continuation of, Mr. Sauerbeck's figures. The same commodities are used, and the Index Number for the complete series is prepared in exactly the same way as that described above, but for the sake of additional clearness and giving additional information the commodities are grouped in the manner already mentioned above. The groups are composed as follows:—

**1. Foodstuffs.**

(*a*) *Vegetable Foods.*—Commodities numbered 1 to 8 inclusive on the above list.

(*b*) *Animal Foods.*—Commodities numbered 9 to 15 inclusive.

(*c*) *Sugar, Coffee and Tea.*—Commodities numbered 16A to 19B inclusive.

*Total of Commodities in Group* = 19.

**2. Materials.**

(*a*) *Minerals.*—Commodities numbered 20A to 26 inclusive.

(*b*) *Textiles.*—Commodities numbered 27 to 34 inclusive.

(*c*) *Sundries.*—Commodities numbered 35A to 45B inclusive.

*Total of Commodities in Group* = 26.

A separate Index Number is compiled for each group, and also for each class in the group, and thus any change of price in any class or group can easily be seen, since the commodities so classified are those which are likely to be affected by any factor.

## § 8.—The "Economist's" Index Number.

This well-known Index Number was originally compiled from the prices of twenty-two articles in common use, grouped in the manner mentioned above. The base period was composed of the years 1845 to 1850, and the average price of *each* commodity for that period was equated to 100, and *the sum of the percentages* was the Index Number. The base Index Number was therefore 2,200.

Owing to changing conditions and the increasing use of commodities other than those originally taken, the Index Number was revised in 1911, and the number of examples increased to forty-four, some commodities being given more than one quotation, thus weighting them according to the extent of their use The base was also changed to the average prices ruling during the period 1901 to 1905. The sum of the base equations would therefore be 4,400, but in order to simplify comparisons this was divided by two and the same base of 2,200 obtained.

To obtain the current Index Number the average price for the current period is converted into a percentage of the base price, and the whole of the percentages added together and divided by two. The Index Number can therefore be easily converted into an average by dividing the sum of the percentages by the number of commodities.

## § 9.—The Board of Trade Index Number.

This Index Number is computed from data relating to forty-seven articles or groups of articles. Most of the data does not relate to actual prices, but to the average declared value of the goods included as imported to or exported from the United Kingdom. The averages taken are average import values in thirty-five cases, and export values in four, the others being obtained from official sources. The Index Number is a weighted one, and the base period is 1900. At the present time the question of revising the method of its computation is under consideration, and efforts are being made to overcome many of the objections to which Index Numbers are open.

## § 10.—Index Number of the Course of Wages.

Prior to the War the Board of Trade compiled an Index Number to show the changes which took place in Wages. The Course is divided into five groups, viz.: Building, Coal-mining, Engineering, Textile and Agriculture, and the average wages of selected examples in each trade during the year 1900 was taken as the base, and each equated to 100. The average ruling rates for similar examples were then converted into percentages of the base year's average, and thus an Index Number for each trade was obtained. To get a General Index Number, the arithmetic average of the Sectional Index Numbers was taken. No figures are as yet available for a period later than 1913.

## § 11.—Use of Index Numbers for Purposes of Comparison.

The following Table extracted from the *Statist* shows how the Index Numbers can be used for the purposes of comparison:—

ANNUAL INDEX NUMBERS FOR A PERIOD OF SEVEN YEARS.

| Class. | 1919. | 1918. | 1917. | 1916. | 1915. | 1914. | 1913. |
|---|---|---|---|---|---|---|---|
| Vegetable Foods | 176 | 171 | 177 | 133 | 108 | 75 | 69 |
| Animal Foods .. | 214 | 207 | 192 | 152 | 126 | 100 | 99 |
| Sugar, Coffee and Tea .. .. | 148 | 138 | 113 | 86 | 70 | 58 | 54 |
| FOODSTUFFS .. | **184** | **177** | **169** | **130** | **107** | **81** | **77** |
| Minerals .. | 210 | 187 | 172 | 158 | 126 | 99 | 111 |
| Textiles .. | 228 | 222 | 192 | 129 | 92 | 81 | 84 |
| Sundries .. | 223 | 202 | 174 | 136 | 109 | 87 | 83 |
| MATERIALS .. | **221** | **204** | **179** | **140** | **108** | **88** | **83** |
| TOTAL .. | **205** | **193** | **175** | **136** | **108** | **85** | **85** |

The Index Number shown for the two main groups, as well as that shown for the Total, are not, of course, obtained from the figures of the classes in the group, but are obtained separately by averaging the total of the indices of all the commodities in the groups or in the Total.

It will be seen from the above Table that though prices have increased considerably since 1913, the change has not been uniform throughout all the commodities, for while Vegetable Foods in 1919 show an increase of 155 per cent. over the prices ruling in 1913, Animal Foods show only a rise of 116 per cent. in the same period, while Tea, Sugar and Coffee have an upward movement of 175 per cent. on the prices ruling for the same commodities in 1913. Foodstuffs as a whole show an increase of 138 per cent. In the second group we find that Minerals have risen in price by only 89 per cent. on 1913, but Textiles and Sundries show increases of 171 and 168 per cent. respectively, and hence the advance in the price of materials in the war years has been 142 per cent. of that in 1913. The total shows that the general level of prices as shown by the articles specified above has increased since 1913 by 141 per cent. To put the matter more plainly, it means that those commodities which were obtainable in 1913 for £100 now cost the consumer £241, a very material difference.

## § 12.—Imperfections of Index Numbers.

To the average man, however, it is no compensation whatever to know that while he has to pay 174 per cent. more for his Tea, Sugar or Coffee, he only pays 89 per cent. more for those Mineral commodities which he uses. Hence, though such an Index Number as this shows the general trend of the prices, it is by no means correct or conclusive, for though the commodities chosen are those in common use, the assumption is made that an equal number of units of all the commodities is available, or is demanded, and it by no means follows that such is the case. Another unsatisfactory feature of such an Index Number is that no consideration is paid to the quality of the articles, and this varies very greatly. Obviously, an article of better quality will cost more than an article of inferior quality, but frequently our sources of Statistics pay no heed to this important point. An example of this is to be seen in the "Statistical Abstract of the United Kingdom," where the average price of Rubber is calculated from the prices of all kinds of rubber, including Waste and Re-Claimed Rubber. We are compelled, therefore, to ask ourselves to what extent the change in the quality of the commodities has affected the price. Unfortunately, the published Statistics as compiled by the various Government Departments do not give us this information, and consequently if we desire to estimate the effect of such changes in quality it is necessary that we carefully record reliable market reports of the various qualities, and thus after the lapse of a long term we may be enabled to gauge the effect of such changes, though even then the information will be largely of an indefinite nature, since so many changes in quality are possible.

## § 13.—Cost of Living Index Number.

Where an endeavour is being made to obtain an indication of the effect the change of prices has upon the cost of living, an entirely different basis must be used. In the first place the prices upon which we are going to work must be those ruling at the point of consumption, that is to say *Retail* prices must be taken. So far we have only dealt with wholesale prices, or what amounts to more or less the same thing, the import or export value of the goods. Wholesale prices do not vary greatly anywhere within the confines of the country, and thus are more suitable when we are desirous of obtaining a reliable guide to a change in price, but these prices bear little if any actual relation to the price which the actual consumer pays. In using retail prices we are met with the difficulty that these vary in different localities, that is apart from proprietary brands of articles which are usually the same in the different districts of the same country. An article purchased in Bond Street, for instance, is likely to cost more than an exactly similar article purchased in, say, the East End of London. While this is largely due to the dictates of fashion and a wealthier class of purchaser, the heavier establishment charges are also in a measure responsible. It must also be borne in mind that when goods have to be transported considerable distances the cost is increased, and hence people living at a considerable distance from the source of supply must of necessity pay more for their goods than those residing at, or near, the place of production. Another difficulty that we have to contend with in compiling such an Index Number is the varying standards of living which exist, for not only is the monetary standard different, but the articles consumed vary greatly also, only a few staple commodities such as Bread and Potatoes being found amongst the commodities consumed by all classes. The quantities of commodities consumed by various classes also show considerable variation. If these difficulties are to be thoroughly overcome it would be necessary to construct a separate Index Number for each separate class, and, even where this was possible, the number of examples which did not fall into the different classes would be very great.

The principal effort to construct a **Cost of Living Index Number** is that of the Ministry of Labour, and this has been the basis of many of the recent claims for increased Wages made by the Trade Unions of this country. It has been designed to measure *the average increase in the cost of maintaining the Pre-War Standard of living of the Working Classes.* The following description of the methods of compilation shows how many of the difficulties have been overcome, and this has been obtained from a description of the process which appeared in the *Labour Gazette* for March, 1920.* The items included in the Statistics fall into five main groups, viz.: (1) Food; (2) Rent; (3) Clothing; (4) Fuel and Light; and (5) Sundries.

### 1. Food.

*Collection of Information.*—The foodstuffs included are Beef, Mutton, Bacon, Fish, Flour, Bread, Potatoes, Tea, Sugar, Milk, Butter, Margarine, Cheese and Eggs. These normally account for over three-fourths of the total family expenditure on food, and the inclusion of a greater number of articles of relatively minor importance would not materially affect the average percentage increase. The most important omission is that of fruit and vegetables (other than potatoes), which it is impracticable to include in a series of price index numbers, owing to wide variations in quality, the "seasonal" variations in supplies, and the consequent impossibility of obtaining continuous and comparable records of prices.

Information of the predominant retail prices of the articles of food included in the survey is collected at the beginning of each month by the managers of the Employment Exchanges and Branches from representative retailers (including Co-operative Societies, large "Multiple" shop firms, and private shopkeepers) conducting a working-class trade. Information is obtained in all towns with a population exceeding 50,000 at the census of 1911, and also in a representative selection of 540 smaller towns and villages distributed throughout the United Kingdom. Altogether 630 Towns and Villages are reported upon. The total number of retailers applied to is about 5,500, but this is not a full indication of the basis of the returns, as many of these retailers have a number of shops at which the prices charged are identical. In some cases also the prices are regulated by the Food Controller or by arrangement among traders, so that an article is sold at a uniform price throughout a locality, the price being known to everyone.

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Great care is exercised in the tabulation of the results of the inquiries, the columns are then added, and the average price of each article is arrived at, separately for large Towns (i.e. those with populations over 50,000) and for several of the Metropolitan districts, and for smaller places, by taking the simple arithmetical average. A calculation is then made of the percentage increase in the price of each commodity, as shown by comparing the average price on the current date with that for July 1914. The percentage for the United Kingdom as a whole is obtained by taking the average mean (*i.e.* the half-way point) between the percentage for the large towns and that for the smaller places.

Having thus arrived at the average percentage increases in the prices of each separate article of food, it is necessary to combine these percentages so as to obtain a single figure representing the general average increase in food prices. The individual percentages vary considerably, beef prices for example having little more than doubled, while granulated sugar is about four times the pre-war price. As very different proportions of pre-war income were spent on beef and sugar, it would not be correct simply to add together the percentages shown for these various items and divide by the number of items (*i.e.* extract the arithmetic average). It is necessary to allow for the difference in the importance of each item in the pre-war budget, and this is done by multiplying each percentage by a number based on the relative importance of the several articles, and dividing the product by the sum of these numbers (*i.e.* a weighted average is extracted). The weights used are based on the average expenditure shown by 1,944 urban working-class budgets collected by the Board of Trade in 1904. The adoption of weights based on the expenditure of 1904 was not invalidated by changes in quantities between 1904 and 1914, because the changes in quantity (which were generally slight) were largely counterbalanced by changes in price-level, with the result that the proportions of income spent on the different commodities could not have been sensibly altered except as regards margarine, for which a special allowance was made. The particulars for the several items are given in the following Table, in which the last two columns illustrate the Arithmetic process by which the general average for *Food* is computed from the percentage increases for the several items. The percentages given are those obtained from the returns relating to March 1, 1920.

| 1<br>Article. | 2<br>Average Expenditure on These Articles in Budgets of 1904. | | 3<br>Weights Proportional to Such Expenditure. | 4<br>Percentage Increase up to March 1, 1920. | 5<br>Result of multiplying 3 and 4. |
|---|---|---|---|---|---|
| | s. | d. | | | |
| Beef .. .. | 2 | 5½ | 48 | 114 | 5,472 |
| Mutton .. .. | 1 | 2¾ | 24 | 110 | 2,640 |
| Bacon .. .. | | 11½ | 19 | 156 | 2,964 |
| Fish .. .. | | 5½ | 9 | 113 | 1,017 |
| Flour .. .. | 1 | 0¼ | 20 | 52 | 1,040 |
| Bread .. .. | 2 | 6¾ | 50 | 63 | 3,150 |
| Tea .. .. | 1 | 1½ | 22 | 89 | 1,958 |
| Sugar .. .. | | 11¾ | 19 | 290 | 5,510 |
| Milk .. .. | 1 | 3¼ | 25 | 197 | 4,925 |
| Butter .. .. | 2 | 1½ | 41 | 154 | 6,314 |
| Cheese .. .. | | 6½ | 10 | 130 | 1,300 |
| Margarine .. | — | | 10 | 93 | 930 |
| Eggs .. .. | 1 | 0 | 19 | 229 | 4,351 |
| Potatoes .. .. | | 11 | 18 | 162 | 2,916 |
| Total .. .. | 17 | 1¾ | 334 | — | 44,487 |

Weighted Average Increase = 44,487 ÷ 334 = 133 per cent.

The Total of the 1904 budget includes an estimate of 6d. for margarine. The total expenditure shown relates, obviously, only to the articles specified in the above list. For all items of food the average expenditure in the 1904 budgets was 22s. 6d.

### 2. Rent.

The Rent Restrictions Acts were still in force at the time this Index Number was compiled, and these Acts provided that, for the same accommodation, rents of working-class dwellings could not be raised by more than the equivalent of the increase in rates. Information as to these increases in rates and their effect on rents has been obtained from time to time by inquiries from the Town Clerks of 100 of the principal towns. The information relates to unfurnished dwelling-houses of the type usually occupied by working-class families, and account has been taken of the increased rents due to increased rates.*

### 3. Clothing.

In order to estimate the increases in the price of clothing information is obtained as to the movement of retail prices of men's suits and overcoats (ready-made and bespoke), woollen and cotton material for women's outer garments, woollen and cotton underclothing and hosiery, and boots. The information relates to those descriptions of articles which are most generally purchased by the working classes, *i.e.* to relatively low-priced grades.

An inquiry form is posted each month to a selection of representative outfitters, drapers, and boot retailers in all the large towns in Great Britain. Before the form is despatched to a retailer the prices quoted by him at the previous inquiry are entered upon it, and he is asked to quote the current price for the same article and quality as before, or for the most nearly corresponding article or quality. The descriptions of articles for which quotations are given vary with different retailers.

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* As Rents may now be increased, this part of the work will need revision if the Index Number is to represent anything like the true state of affairs.

For other commodities, *e.g.* foodstuffs, the predominant prices were averaged, and this average forms the basis of the final figures. For articles of clothing, however, the range of prices and qualities is so wide that this method is unsuitable, and the method adopted, therefore, is to compute a separate percentage increase for each quotation supplied by each retailer, to tabulate these percentages, and to combine them for each article by taking the simple arithmetic average of the percentages. The resulting figures are combined into groups of similar kinds of articles, by averaging, and these group figures are then averaged to give the general figure.

It will be observed that part of the information collected relates to the prices of materials for clothing instead of to the articles of clothing themselves, because for the former alone could comparative prices be obtained over a period with a fair measure of accuracy. In times when prices were moving slowly such figures could be held to represent fairly the increase in the prices of the articles of clothing, but recent increases in the prices of textiles have been so great that in utilising the prices of materials, to obtain an estimate of the amount of the increase in the price of clothing, an allowance is made for the fact that the cost of making up has not advanced similarly. Of course, where garments are made up at home the increase in the cost of the material closely represents the increase in the cost of the garment.

It will be recognised that owing to the wide range of quotations, both now and before the war, to changes in qualities and in the stocks held by the retailers, and to variations in the extent to which different articles and qualities have been affected by price changes, it is not possible to calculate with precision the average percentage change in prices of clothing, and the percentage indicated by the method of averaging described above can only be regarded as an approximation.

### 4. Fuel and Light.

The items included in this group are Coal, Gas, Oil, Candles and Matches. Statements are obtained each month from correspondents as to the prices of coal in thirty of the principal towns, and of gas in twenty-six of the principal towns. The figures obtained in this way are supplemented from time to time by information from all the large towns. Returns as to the prices of oil, candles, and matches are obtained each month direct from retailers in twenty-eight of the principal towns. In all cases the correspondent or retailer is supplied with the prices returned at the previous inquiry, so as to ensure that the quotations are comparable.

The prices quoted in the returns are tabulated and scrutinised, and an average price is then arrived at for each of the five articles by totalling the quotations and dividing by the number of items. A computation is then made of the percentage increase shown by the average price of each article over the average price of the same article in July 1914. The percentage increases so obtained are combined into a general average for fuel and light by weighting them in accordance with their approximate relative importance in pre-war working-class family expenditure, the proportions taken being 6 for coal, 3 for gas, and 1 for oil, candles and matches together; these figures being equivalent to pre-war average expenditures of 1s. 6d. to 2s. per week on coal, 9d. to 1s. on gas, and about 3d. on oil, candles and matches.

### 5. Other Items.

The items included in the Statistics under this heading are soap and soda; domestic ironmongery, brushware and pottery (as indicated by the prices of saucepans, kettles, frypans, bowls, pails, scrubbing brushes, banister brushes, brooms, cups and saucers, plates, basins, dishes, jugs and teapots); tobacco and cigarettes; fares; and newspapers. Of these the prices of soap and soda are ascertained each month from shopkeepers in twenty-eight of the principal towns, and those of ironmongery, brushes and pottery from shopkeepers in twenty-two of the principal towns. Particulars of the changes which have occurred in the prices of tobacco and cigarettes, fares and newspapers are mostly available through the medium of public announcements, and when necessary these have been supplemented by special inquiries. The increases so ascertained for each item have been combined, after consideration of the relative importance of the different items, so as to enable the Ministry of Labour to form an estimate of the average increase in prices of these items, taken as a whole, and its effect on the general percentage increase for all items. There is considerable variation both in the total amount of expenditure and in the distribution of the amount among the several items, but the importance of these items is small relative to the total family expenditure, so that the precise amount of increase is not a matter of substantial importance in its influence on the general result.

### Combination of all Items Included.

In order to arrive at a single figure, representing the increase since July 1914, in the prices of all the items taken together, the average percentage increases under each of the main groups of expenditure referred to above are combined in accordance with their estimated relative importance in average pre-war working-class expenditure, the weights used being as follows: Food, 7½; Rent (including Rates), 2; Clothing, 1½; Fuel and Light, 1; Other Items included, 1. In other words, the percentage increase ascertained for Food is multiplied by 7½, that for Rent by 2, and so on with the other groups; the results are added together, and the total is divided by 12½ (the sum of the weights). The effect is to obtain the average percentage increase in the cost of maintaining the pre-war standard of living.

With regard to the basis of these weights, the budgets collected in 1904, to which reference has already been made, showed that, on the average, 22s. 6d. out of a family income of 36s. 10d. per week was spent on Food, or about three-fifths of the total. Between 1904 and 1914 there were almost equal increases, on the average, in prices and wages, and it is therefore unlikely that this proportion would have altered appreciably during that period; the total weight allotted to food is therefore three-fifths of the total of 12½, *i.e.* 7½.

Information as to Rents, which was obtained in the course of the Inquiry into the Cost of Living in 1912, showed that the *average* working-class rent in the industrial towns was about 5s. 6d. to 6s. per week. (It should be noted that in London the rents were considerably higher than the average, whilst in other towns they were appreciably lower.) In 1914 it was a little higher, probably about 6s. per week. That is to say, the proportion of the total family income which was expended on rent amounts to between one-sixth and one-seventh, and the weight taken for rent is accordingly 2 out of a total of 12½.

As regards clothing, there are wide variations in the expenditure of different sections of the working classes, but pre-war investigations showed that, on the average, expenditure on clothing before the war was less than that on rent, and in many cases much less. In the absence of exact data, a weight of 1½ out of the total of 12½ is taken for clothing, on the basis of the estimated pre-war expenditure by working-class families on clothing, being on the average 4s. 6d. to 5s. per week.

For Fuel and Light, also, there are no extensive statistical data as to expenditure before the war, but such information as is available indicates that the average expenditure was probably from 2s. 9d. to 3s. per week, which yields a weight of 1 out of a total of 12½. The weight of ½ for the other items included was taken as fairly representing the average expenditure on these particular commodities.

### Items not Included in the Statistics.

The list of items included in the Statistics is considered sufficiently extensive and representative to provide a sound basis for estimating the average increase in the cost of maintaining the pre-war standard of living of a working-class family. As regards items which are not included, it may be observed that they form in the aggregate only a very small proportion of the total weekly working-class expenditure, and that some have increased more, and some less than the average. Consequently it is very unlikely that the general average increase would be appreciably affected by the inclusion of a larger number of items, even if it were found practicable to extend the list. It will be evident that if a particular item of expenditure had risen by about the same amount (130 per cent.) as the items included in the statistics, its omission or inclusion would have no effect on the general percentage, and that the omission of certain small items of expenditure as to which reliable statistics are not obtainable would only become important if the *average* percentage rise in the price of these omitted items (taken together) were either very much below or very much above the general average.

### Why such a Number is Unsatisfactory.

It is apparent from the above description that the computation of a Retail Price Index Number calls for considerable work in obtaining the necessary data upon which to work. In making the Index Number one which deals with the cost of living many difficulties have to be overcome, and even then the result is far from reliable, and should be used as a guide to the percentage increase only. Even where the total weekly expenditure was the same in different families, however, the distribution of the expenditure over different commodities varied considerably according to the numbers of persons in the family, their ages, sex, and mode of living; and as some articles have risen less in price than others, even the percentage increase would vary to some extent with different families. Thus, bread, margarine and tea have risen less in price than the average, and the percentage increase of the cost of maintaining a dietary which consisted mainly of those articles is, therefore, less than that of maintaining a dietary consisting mainly of eggs and milk, the prices of which have risen much more than the average. It is obvious from this that no single figure with regard to the increase in the cost of living can apply to every individual case in the class of family dealt with in the Statistics.

It will be observed that, with the exception of margarine, no allowance has been made for any change in consumption during the period which has elapsed since the budgets were gathered in 1904. This we know is not the case, for, during a part of the war period, there was such a shortage of some commodities that some were difficult to obtain at any price, or alternatively were rationed to everyone in small quantities. Without doubt the habits of people were somewhat affected by this, and thus it may be found that the quantities of various articles consumed, and therefore the proportion of expenditure on these articles, has changed materially. If such is the case, the Index Number does not represent the true state of affairs, and thus any conclusions or arguments drawn from it would be erroneous.

## § 14.—Commercial Uses of Index Numbers.

### (a) The Wages Problem.'

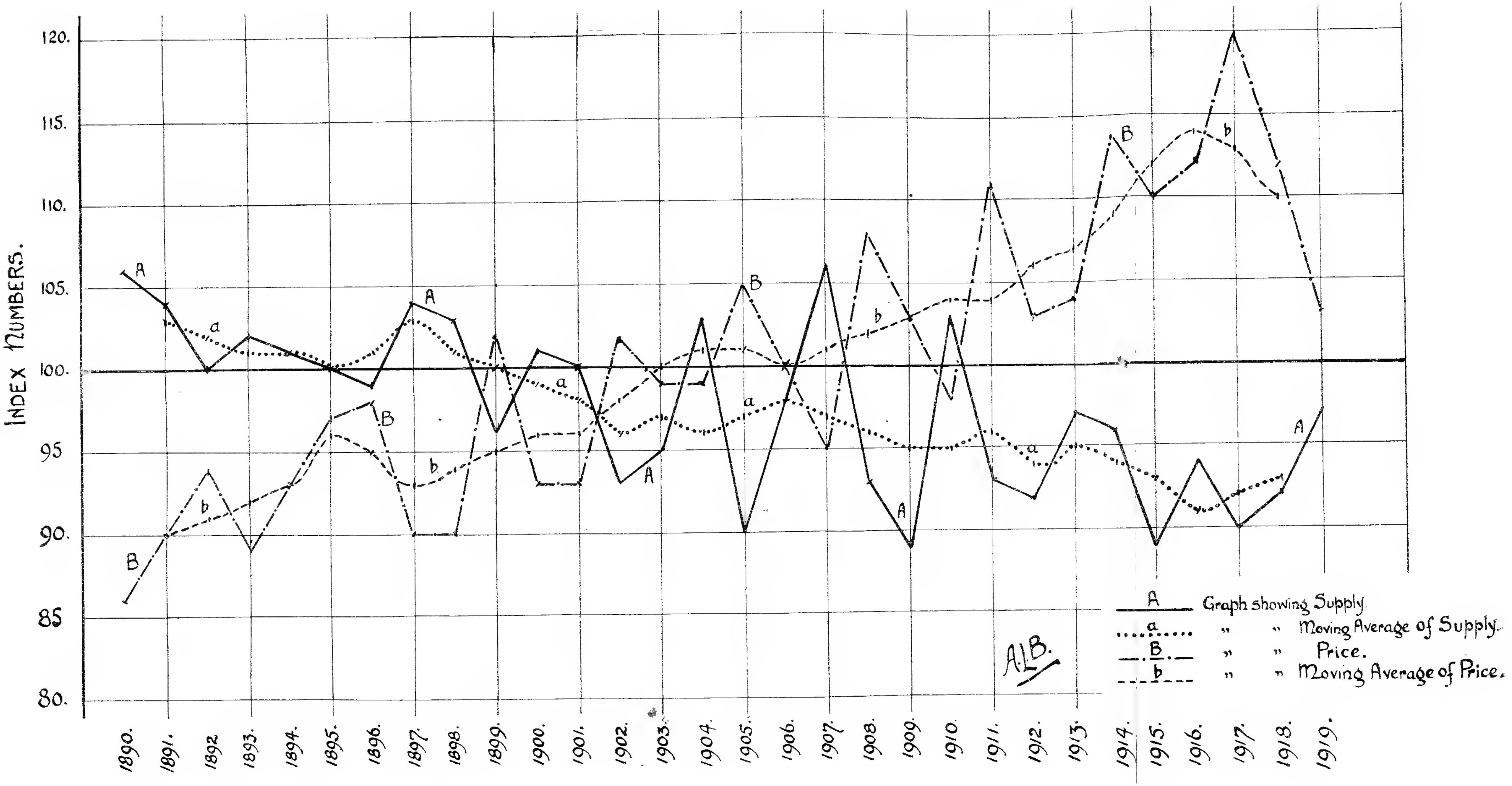
Whatever the merits or demerits of this Cost of Living Index Number, it has been the basis for adjustments of wages in various trades in this country, as evidenced by the special bonus to the Police in September, 1920, to meet the increased cost of living. Our American friends, however, have adopted a method of paying bonuses which vary with the cost of living, which method shows a distinct advantage over the system in vogue in this country. Here the adjustments have been made after some time has elapsed, thus allowing unrest at the increasing prices to develop. The Americans have no special Index Number to represent the increase in the cost of living, but base their calculations upon one or other of the five Index Numbers of Wholesale Prices which are in existence over there. Two examples may serve to show the method of working their bonus scheme. In the first case a standard rate of wages is fixed, and the Index Number ruling at the date of the fixing of such standard is the basis of the calculations. The number of points which it is necessary for the Index Number to advance in order to show an increase of 1 per cent. on the price is computed, and when the monthly index number is published, the percentage by which prices have advanced is calculated, and a bonus equal to the same percentage of the wages is then paid to the workers to meet the increased cost of living. The bonus thus follows the increased cost very closely indeed, and the worker knows that any increase of the price of commodities will be reflected in the amount of his bonus at the end of the month. For example, assuming a worker is earning 30 dollars a week as his standard wage, and the Index Number has to show an advance of 5 points over the base Index Number at the time of fixing the Standard to give a 1 per cent. increase on cost, then if in any month the Index Number shows an upward movement of 10 points, the worker would receive a bonus of 2 per cent. of his wages, *i.e.* 2 per cent. of \$30 = 60 cents, as an addition to his week's wages. This bonus is thus payable monthly in arrear. The other method adopted is as follows: Again, supposing the employee is receiving 30 dollars per week, and the Index Number at the time of fixing the standard is 300. If in any month the Index Number shows an increase of 10 points, *i.e.* moves to 310, then the increase is expressed as a fraction of the base Index Number, and the rate of wages is multiplied by the result, *e.g.* in the above case we should get $\frac{10}{300}$ of \$30 = \$1, and this is the amount of the weekly bonus which will be paid to the workman at the end of each month. If the Index Number in the following month shows a fall to, say, 305, the weekly bonus will then be $\frac{5}{300}$ of \$30 = 50 cents. By this means the standard wage is not lost sight of, and the workers' bonuses follow very closely the rises which take place in those commodities which he must buy in order to live. In this country the cost of living bonus is merged into the wages, and thus the standard wage is lost sight of, and, moreover, the bonus does not vary with the fluctuations in the cost of commodities, but is arbitrarily fixed from time to time.

### (b) Correlation of Factors.

It frequently becomes necessary, particularly in commerce, to consider two sets of data which are correlated, with a view to ascertaining the degree of correlation which exists. If we take two such factors as the supply and price of a commodity (or commodities) and desire to ascertain the effect which the supply has upon price, we must consider the data from two aspects. We must, firstly, eliminate the short-term fluctuations and consider the trend of both price and supply, and secondly we must eliminate the long-term fluctuations in order to consider the effect that abnormal conditions in one factor have upon the other at the time the condition rules. We have already seen that we can eliminate the short-term fluctuations of any single series of data by plotting the moving average line, and can then measure the effect of abnormal conditions by comparing the curve of actual figures with the curve of the moving average. If reference be made to Diagram No. 8, the fluctuations from the moving average line or trend can be very clearly seen, and the difference can easily be measured. As we are already aware, these short-term fluctuations have no marked effect upon the trend, as may be seen from the regularity of the upward movement in the moving average line in the diagram mentioned, though the actual figures were subject to wide variations. When comparison between two sets of data is necessary, the data to be compared should be placed upon the same chart, but as different units and types are being used, great care has to be exercised in plotting the data if the true comparison is to be found. This difficulty can be eliminated by using Index Numbers for each of the factors, as they will then be reduced to similar terms and can be plotted on the same scale. Columns 1 and 4 of Table X (on opposite page) show the Index Numbers of the Supply and Price respectively of any commodity. The figures are entirely hypothetical, and have been compiled for the purpose of illustration only. Columns 2 and 5 show the three-yearly average of the Indices so gathered. The figures in these four columns have been plotted in Diagram No. 23, where the curve A shows the actual indices of the Supply of the Commodity, and curve *a* the moving average of the indices or the trend of the supply. The other curves marked B and *b* show the Indices and the averages respectively of the price. The curves *a* and *b* can be compared with a view to discovering if there is any relation existing

## DIAGRAM No 23.

Diagram (with Moving Average Line) showing Relation between SUPPLY and PRICE.



between the two over the period under review. If the curves move together either in the same or opposite directions, then some degree of relationship apparently exists

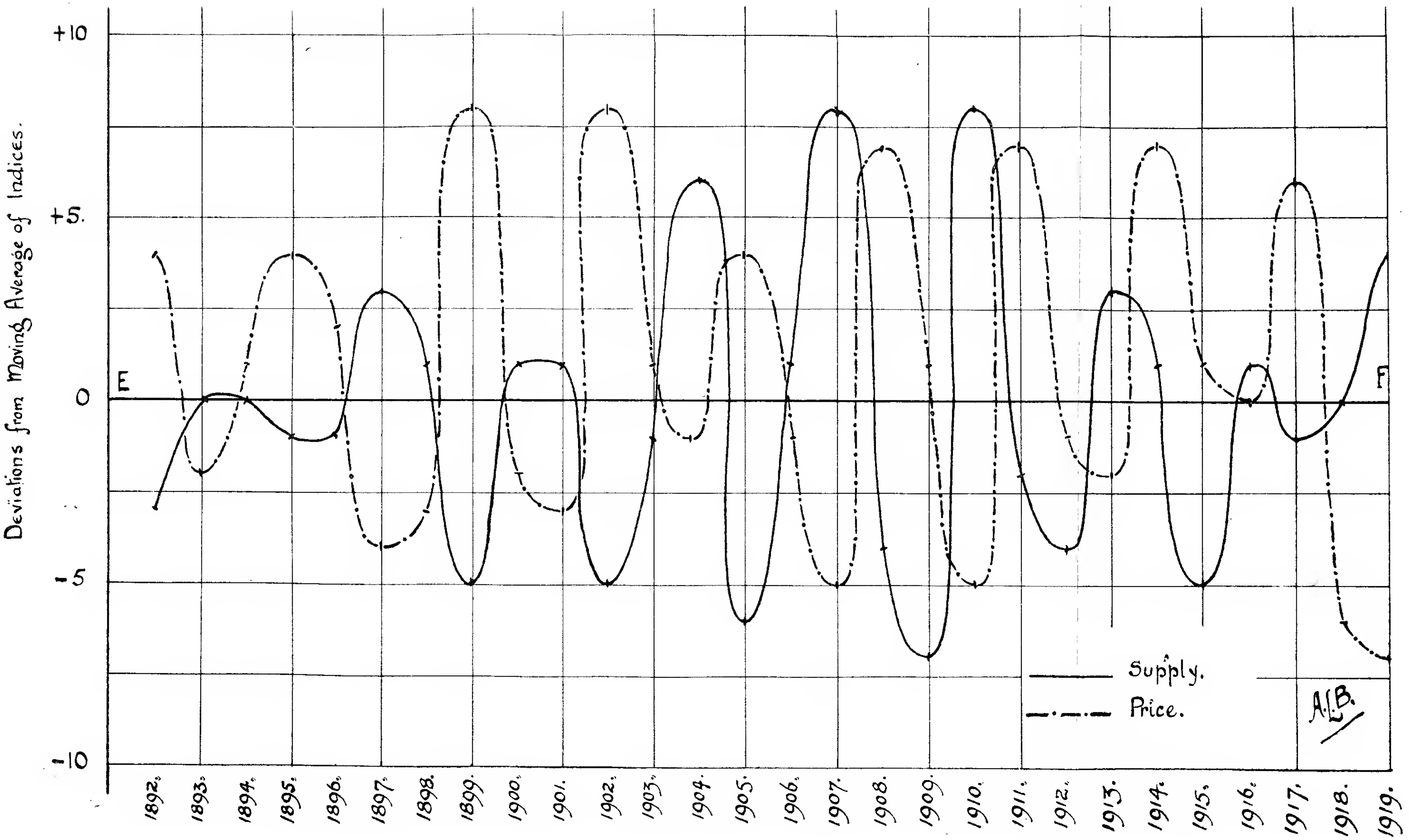
TABLE X.

| Year. | Supply. | | | Price. | | |
|---|---|---|---|---|---|---|
| | 1 Index Numbers. | 2 Moving Average of Indices. | 3 Deviation from Moving Average. | 4 Index Numbers. | 5 Moving Average of Indices. | 6 Deviation from Moving Average. |
| 1890 | 106 | — | — | 86 | — | — |
| 1891 | 104 | — | — | 90 | — | — |
| 1892 | 100 | 103 | − 3 | 94 | 90 | + 4 |
| 1893 | 102 | 102 | 0 | 89 | 91 | − 2 |
| 1894 | 101 | 101 | 0 | 93 | 92 | + 1 |
| 1895 | 100 | 101 | − 1 | 97 | 93 | + 4 |
| 1896 | 99 | 100 | − 1 | 98 | 96 | + 2 |
| 1897 | 104 | 101 | + 3 | 91 | 95 | − 4 |
| 1898 | 103 | 102 | + 1 | 90 | 93 | − 3 |
| 1899 | 96 | 101 | − 5 | 102 | 94 | + 8 |
| 1900 | 101 | 100 | + 1 | 93 | 95 | − 2 |
| 1901 | 100 | 99 | + 1 | 93 | 96 | − 3 |
| 1902 | 93 | 98 | − 5 | 102 | 96 | + 8 |
| 1903 | 95 | 96 | − 1 | 99 | 98 | + 1 |
| 1904 | 103 | 97 | + 6 | 99 | 100 | − 1 |
| 1905 | 90 | 96 | − 6 | 105 | 101 | + 4 |
| 1906 | 98 | 97 | + 1 | 100 | 101 | − 1 |
| 1907 | 106 | 98 | + 8 | 95 | 100 | − 5 |
| 1908 | 93 | 97 | − 4 | 108 | 101 | + 7 |
| 1909 | 89 | 96 | − 7 | 103 | 102 | + 1 |
| 1910 | 103 | 95 | + 8 | 98 | 103 | − 5 |
| 1911 | 93 | 95 | − 2 | 111 | 104 | + 7 |
| 1912 | 92 | 96 | − 4 | 103 | 104 | − 1 |
| 1913 | 97 | 94 | + 3 | 104 | 106 | − 2 |
| 1914 | 96 | 95 | + 1 | 114 | 107 | + 7 |
| 1915 | 89 | 94 | − 5 | 110 | 109 | + 1 |
| 1916 | 94 | 93 | + 1 | 112 | 112 | 0 |
| 1917 | 90 | 91 | − 1 | 120 | 114 | + 6 |
| 1918 | 92 | 92 | − 0 | 107 | 113 | − 6 |
| 1919 | 97 | 93 | + 4 | 103 | 110 | − 7 |

in the trends. It will be seen in this case that even after eliminating the short-term fluctuations the line showing the trend of supply moves in the opposite direction to the curve for the moving averages of the indices of price. There are minor differences, as is only to be expected, but the general tendency of the curves is in opposite directions, thus indicating that price recedes where supply advances, and *vice versa*. If we now eliminate the long-term fluctuations and compare only the short-term variations, we may obtain other information of great value. In columns 3 and 6 are tabulated the deviations which the index for the year shows from the moving average or trend, and these deviations have been plotted in Diagram No. 24. The zero line (marked EF) may be regarded as the straightened moving average lines in the previous diagram, and the curves show how the indices for supply and price deviate from this line. It will at once be seen that in nearly all cases when supply is low, prices are high, and *vice versa*, but in 1912 and 1915 the deviations appear to be in the same direction. The deviations vary considerably in magnitude, *e.g.* in 1899 and again in 1902 an equal fall in the supply was accompanied by rises in price which were identical in each case, but in 1905, though the supply was even shorter, the price did not rise so sharply as in the other two years. In several years the highest rise in one exactly corresponded in time with the lowest of the others (*c.f.* 1897, 1899, 1902, 1905, 1919), but in a few cases there was a " lag " in one of the factors. In 1908, for example, the price deviation was high, and then fell away in the following year; but though supply fell in 1908, it was lower still in 1909. Again, in 1911 the price deviation was high and that for supply low, but in 1912 both price and supply receded. These circumstances indicate that other factors, such as speculation, were influencing the price, but it is clear that the supply is the main factor regulating price. Knowing this is so in any particular case, the astute business man in possession of information as to the likelihood of supplies being short would anticipate the rise in price by buying early. This is

## DIAGRAM No 24.

Diagram showing SHORT TERM FLUCTUATIONS in SUPPLY and PRICE.



where valuable information is given by commercial newspapers in supplying estimates of crops, output, etc.

### (c) Investment Index Number.

That Index Numbers can be utilised in Commerce can be open to no doubt, for unlike the objections to the types already dealt with, the information in a business house is on much more definite lines. The British, Foreign and Colonial Corporation, Limited, have compiled an Investment Index Number which should be of great assistance to those interested in the Stock Markets. Stocks and Shares numbering in all 100 were grouped into five classes, viz: (1) Gilt-Edged; (2) Investments carrying Fixed Interest; (3) Investments with Varying Interest; (4) Semi-Speculative Securities; and (5) Speculative Stocks and Shares, and each group was composed of selected representative securities.

The basis of the number was the capital and market quotations on December 31, 1918, and the average prices of each group was equated to 100. The average market value at any date can then be compared with the base figure, and the percentage of increase or decrease ascertained. This is a straightforward method when the capital in the various undertakings remains unchanged, but where there is a fresh issue of capital either for cash or in the form of bonus shares, either free or at a reduced price, these changes in the capital must be taken into consideration in order that a correct comparison of any month's figures is possible. If any Company whose Stocks or Shares are included in the list is liquidated, or bought out, a fresh security as similar in all respects as is possible has to be substituted and the capital involved adjusted accordingly. The following copy of the monthly report of the Corporation (inserted by their kind permission) shows how the data obtained are utilised for advising their many clients of the general movement of prices in Stock Exchange Securities:—

## INVESTMENT INDEX FIGURES FOR 1920.

The Investment Index Figures of the British, Foreign and Colonial Corporation, Limited, show that after making the necessary adjustments for bonus distributions, new issues of capital, etc., **100 standard investment securities** of an aggregate value on December 31, 1918, of £2,991 millions, and on December 31, 1919, of £3,031 millions, had a market value on December 31, 1920, of **£2,615** millions, a decrease of 12·6 per cent. on the quotations on December 31, 1918, and a decrease of £27 millions, or ·9 per cent., on the quotations on November 30, 1920.

Taking the 1918 figures as 100, the percentage totals of the different classes of investments are as follows:—

### Monthly Index Figures for 1920.

| Type. | Dec., 1919. | Jan., 1920. | Feb., 1920. | Mar., 1920. | Apr., 1920. | May, 1920. | June, 1920. | July, 1920. | Aug., 1920. | Sept., 1920. | Oct., 1920. | Nov., 1920. | Dec., 1920. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Gilt-Edged .. .. .. .. | 96 | 95 | 92 | 92 | 88 | 90 | 89 | 88 | 88 | 88 | 86 | 86 | 86 |
| Investment (Fixed Interest) .. | 95 | 96 | 95 | 90 | 88 | 89 | 87 | 89 | 91 | 93 | 93 | 92 | 91 |
| Investment (Varying Interest) .. | 110 | 113 | 103 | 100 | 94 | 92 | 89 | 90 | 87 | 87 | 88 | 82 | 79 |
| Semi-Speculative .. .. .. | 129 | 144 | 134 | 124 | 117 | 112 | 108 | 109 | 112 | 114 | 113 | 100 | 95 |
| Speculative .. .. .. .. | 145 | 151 | 149 | 130 | 121 | 115 | 109 | 110 | 109 | 107 | 102 | 89 | 83 |
|  | 101 | 103 | 99 | 97 | 92 | 93 | 91 | 91 | 92 | 92 | 90 | 88 | 87 |

### Features of December, 1920.

The steady decline in values continued throughout the month, but was helped in the last few days by a sudden rally, from which the Gilt-edged and Speculative groups especially benefited. In Fixed-interest-bearing stocks liquidation of South American loans mainly accounts for the fall, whilst **Dunlops**, with a loss of 4·8 millions, and **Shells**, with 5·5 millions, were the heaviest sufferers in the Semi-speculative section.

This Index Number is, of course, open to the objection that an arbitrary period was chosen as the base, for obviously the date chosen (December 31, 1918) was one when we were passing from a state of war to that of peace, and optimistic aspects had somewhat inflated prices. The average prices ruling over a series of years would have been preferable, but it is a step in the right direction, and should assist in presenting the utility of Index Numbers to the general public.

**(d) Costing.**

An Index Number compiled on a similar basis for the costs of production would be extremely useful for purposes of comparison in a manufacturing business. By taking the average costs of the following items for a series of years and equating each to 100—as well as the Total Cost and Total Output—comparison would be easily and advantageously carried out: —

Materials .. .. .. .. ..
Direct Wages .. .. .. .. ..
Indirect Works Charges .. .. ..
Other Indirect Charges .. .. ..

Total Cost .. .. .. .. ..

Total Output .. .. .. .. ..

The item of Total Output would present certain difficulties, in so far that it is extremely unlikely that at the end of any period all the work in hand will be completed. When an efficient Costing System is in operation it will be possible to obtain the actual cost of such unfinished work as exists, and such costs would be deducted from the total figure charged to the various items before the percentage is calculated, thus ensuring that the costs relate only to the actual finished output.

## SYNOPSIS TO CHAPTER IX.

# PUBLISHED STATISTICS AND THEIR USE.

§ 1.—Railway Statistics.
2.—Meaning of Terms.
3.—Method of Compilation.
4.—Comparability of Data.
5.—Completeness of Information.
6.—Period Covered.
7.—Degree of Accuracy.
8.—Making Data Comparable.
9.—Clearness of Presentation.
10.—Correctness of Calculations.

## CHAPTER IX.

# PUBLISHED STATISTICS AND THEIR USE.

THE greatest possible care must be taken in considering and utilising Statistics other than those compiled under one's own supervision. Published tabulated returns, no matter from what source they are obtained, are frequently unreliable, and should be used with discretion unless the method of compilation and the reputation of the compilers are well known.[*] Government Statistics—or those presented to the public by official or semi-official bodies—are open to serious criticism, since up till now there has been no uniformity in the compilation, calculation, tabulation or presentment of the data, each Department collecting, recording and presenting the data independently and according to its own definitions. The Conference of the Government Officers engaged in dealing with Statistics in the British Empire, which was held in London early in 1920, should ensure greater uniformity, and thus greater reliability, than has previously been achieved.

## § 1.—Railway Statistics.

Probably the most scientifically prepared statistics compiled by a Government Department are those on Railway Working presented to the public monthly by the Ministry of Transport, some of which returns have been used to illustrate this book. By compiling statistics relating to such matters as the number of " Train-Miles " run by (*a*) Loaded Trains, (*b*) Empty Trains, and (*c*) Engines with only one or two brake vans attached, it can be ascertained whether the companies are utilising their Rolling Stock to the best advantage, for obviously if the ratio which the two latter bear to the former is high, traffic is not being planned economically, and there is a wastage of power and carrying capacity which in the case of a large railway may considerably affect its profit-earning capacity. The average wagon load shows whether trucks are being loaded to their full capacity, as if not there is again wastage, two or more wagons being used when one would do. In order that no unfair inference may be drawn when bulky but light goods are being carried, the goods are grouped into classes. The average Train Load will indicate whether the power of the locomotives is being used economically, for the expense of running a train lightly loaded will be approximately the same as when it is loaded to its fullest capacity. "Wagon-Miles per hour" supplies information as to the speed with which freight is moved, and a comparison between the figures obtained from the various companies will show at a glance which is the speediest route. "Train-Miles per Engine Hour" shows the actual work performed by the locomotives, and makes allowance for the time which is unproductive, such as preparing the engine for a Run, and in moving it from the shed to the point at which the train will be coupled. "Ton-Miles" is the unit by which one can judge the freight-earning capacity of the Railway. This unit is obtained by multiplying the weight carried by the distance each consignment is conveyed. By taking the figures for various districts an estimate may be made as to the density of the traffic, and also as to varying conditions which may be encountered. The average train load in hilly country cannot, of course, be as high as where the track runs on level ground, but the average wagon load would not need to be altered. The speed would also of necessity be reduced in a populous industrial district, and a larger number of hours spent in shunting than would be the case in an agricultural district.

## § 2.—Meaning of Terms.

In using published Statistics, therefore, we must first inquire as to the exact meaning of the terms used. Tables frequently do not define the units tabulated, thus leading to confusion and needless ambiguity. When we speak of the "cost" of an article, do we mean the Prime, Factory, or Total Cost? If used in the colloquial sense, we may mean the price we paid for it. In dealing with the returns of Companies, we find the word "Capital" is used. What does this term mean? Is it the Authorised or Nominal Capital of the Company which is referred to, or may it be the Subscribed, Issued, Paid-up, or Working Capital which is meant? Until we know exactly the meaning of the term, any conclusions we may draw may be quite erroneous. These are only two examples of many, but they will serve to indicate the importance of the definitions of the units used, and these should always be given. Many controversies would be avoided were the meanings of the terms made clear. We speak of a Company making profits of, say, 100 per cent. of its Capital, and this appears in order when we take the issued and paid-up Capitals as the base of our calculation; but when the Company has built up large reserves out of its profits, and these reserves equal in amount the paid-up Capital, then we find the profits are only 50 per cent. of its working capital—a very different result.

## § 3.—Method of Compilation.

Our second inquiry must be directed to the method of compilation, for if the same definition has not been strictly observed, then the tables cannot be relied upon to give accurate results or deductions. If in the same tables we find examples where different definitions have been used —as when one Company returns its Nominal Capital, and its Paid-up Capital is less than this amount, while another shows its Paid-up Capital—our tables are not homogeneous, and hence cannot be safely used, deductions drawn therefrom being unreliable. It must also be ascertained that every known example has had a chance of inclusion, and that each district—where districts vary in their nature—has contributed its due weight to the total. "Has the method of sampling in different districts been uniform ?" is another question which needs asking. Published Tables seldom give this information, and care must therefore be taken in using them.

## § 4.—Comparability of Data.

The third point to which attention should be directed is whether the data we desire to compare are such as to be capable of comparison. We have already seen that we cannot compare output of goods with the profit of a business, since this latter depends not so much upon the number of articles or quantity produced as on the number or quantity sold. It would obviously be fallacious to compare the average wages ruling in, say, an agricultural with those in a mining district, or the average income of the inhabitants of a fashionable residential centre, with that of the people living in a slum area ; or the output of a factory where everything is done by hand with that of one where the latest labour-saving machinery is installed ; for none of these things are strictly comparable, and mistakes and fallacies will result from comparing such data.

## § 5.—Completeness of Information.

A fourth question will naturally be as to whether the whole of the information is given, for if not fallacious conclusions may be very easily drawn. Our newspapers are apt to err very much in this respect, quoting only those facts which support their arguments and contentions. It would be fallacious, for instance, to argue that because exports were increasing, the trade of the country was in the ascendant, for our foreign trade is only a part of the whole, and the remainder may be suffering from a slump. It is like arguing that because we are suffering from fog the whole world must be in a similar condition. No one with common sense would endeavour to compare the results achieved by various companies if only the actual profits were shown as follows :—

| | |
|---|---|
| Company No. 1 .. .. .. .. | £20,000 |
| ,, ,, 2 .. .. .. .. | 30,000 |
| ,, ,, 3 .. .. .. .. | 35,000 |
| ,, ,, 4 .. .. .. .. | 60,000 |
| ,, ,, 5 .. .. .. .. | 80,000 |

Company No. 5, for instance, made four times the profit of No. 1, and on this basis alone might be accused of profiteering ; but if it were shown that the Capital employed in No. 5 were four times that of No. 1, the matter assumes a different aspect, for though the actual profit in the one case is four times that in the other the rate of profit earned on the Capital involved is the same in each case. Hence actual profits should never be quoted unless supported by other data, such as the capital involved. The percentage rate is preferable. Such a precaution would prevent ambiguity and avoid the making of false or misleading conclusions.

## § 6.—Period Covered.

In the fifth place care must be taken to see that no data in respect of short periods alone are taken and used for the purpose either of comparison or further deduction, since, as we have already seen, these may contain abnormal features which do not make them reliable as a base for further calculations or reasoning. No one would base the sum payable for Goodwill upon the results of any one month's trading—or even on any one year—for neither of the periods is sufficiently long to be a reliable guide to the real value of the business to be obtained.

## § 7.—Degree of Accuracy.

Our sixth inquiry must be as to the degree of accuracy in tabulation and computation, for, as already shown, if we are to carry our calculations further, it is necessary to obtain a greater degree of accuracy than would be the case where comparison alone is the object of our work.

## § 8.—Making Data Comparable.

In the next place care must also be taken to see that the units used are those capable of being compared, as if this is not the case steps must be taken to render them capable of comparison by the use of indices. This is usually necessary when the units of measurements of different commodities vary—*e.g.* production of Wheat is measured in Quarters, while the output of Steel is expressed in Tons—or where the price of one commodity differs widely from the price of another, though the unit of measurement is the same in each case; *c.f.* the price of Coal and Steel where the output of each is measured in Tons.

## § 9.—Clearness of Presentation.

In the eighth place the tables must be examined to see whether the data presented give us the information in the best possible manner. Footnotes must be studied, and where necessary the data must be redrafted on a uniform basis before being used. If the "Statistical Abstract" be studied, many examples will be seen of Tables which do not give information which is desirable. A good example will be found in the figures relating to Bankruptcy, where the data shows the number of bankruptcies, the estimated liabilities and the estimated assets. Such a Table serves no useful purpose, but if the actual value realised from the assets and the amount of the liabilities passed by the Trustee as entitled to rank for dividends had been shown we could have calculated the actual deficiencies. As it is the figures given do not show the actual position, since Debtors are prone to over-estimate the value of their assets and understate their liabilities. A further important improvement would be to show the costs of realisation and the dividend, or dividends, paid to creditors.

### § 10.—Correctness of Calculations.

Ninthly, care must be taken when averages or percentages are given to see that the average used is the one most suitable and that the percentages are calculated on comparable bases. Both these methods of presenting data are loosely used, and when this is the case erroneous conclusions may be drawn.

Where graphic methods are used, the scales should be scrutinised in order to see that a proper relationship exists between the curves shown. The position of the zero line should be verified, or the true perspective will not be obtained, and the various factors plotted should be distinguishable at a glance.

In presenting any statistical data for publication or for presentation to anyone not versed in the technicalities of their preparation, care should be taken to see that these numerous points are covered. When such is done, reliance can be placed upon the data so presented, and by this means the utility of Statistics will be demonstrated and an aid to Scientific Management be available.

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